Comparison of MPC Formulations for Building Control under Commercial Time-of-Use Tariffs

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***Abstract*—Most medium and large commercial buildings in the U.S. are subject to complex electricity tariffs that combine both Time-of-Use (TOU) energy and demand charges. This study analyses the performances of different economic Model Predictive Control (MPC) formulations, from the standpoints of monthly bill reduction, load shifting, and peak demand reduction. Simulations are performed on many simplified commercial building models, with multiple TOU demand charges, and under various summer conditions. Results show that compared to energy-only MPC, the traditional method for dealing with demand charges significantly reduces peak demand and owner bill, however, highlight a lack of load shifting capability. A proposed incremental approach is presented, which better balances the bill components in the objective function. In the case study presented, this method can improve monthly bill savings and increase load shifting during demand response events, while keeping a similarly low peak demand, compared to traditional MPC methods taking into account demand charges.**

***Index Terms*—Model Predictive Control, Time-of-Use tariff, Demand charge, Commercial building, Peak demand**

I. Introduction

The building sector in the U.S. accounts for 40% of country energy consumption [1], and commercial buildings are responsible for 36% of all U.S. electricity usage [2]. The latter is getting increasing attention, due to a high potential for energy saving and load shifting. Constituting a large portion of the energy consumption of commercial building, Heating, Ventilation and Air Conditioning (HVAC) systems represent a primary target of control method research.

Model Predictive Control (MPC) recently emerged as a state-of-the-art method in building energy management [3]. The ability of MPC to take into account future conditions to drive the current system state makes it ideal for Demand Response (DR). DR refers to the set of grid mechanisms to shape the buildings electric load when market prices are high or when the grid reliability is jeopardized [4], through either financial incentives or electricity pricing structures.

Time-of-Use (TOU) tariffs, that define distinct price levels for specific periods of the day, are widely used to incentivize consumers to shift demand outside grid peak hours. In addition, Critical Peak Pricing (CPP) programs superimpose a large increment in energy price to the basic TOU rates for some strategic hours, generally decided a day in advance by the grid utility. This known structure can directly be leveraged by the MPC formulation to shift loads appropriately.

The monthly bill B_m ($) of a commercial building in the U.S. is commonly made of fixed, energy, and demand charges:

$$\mathrm{B}_m = C_{fix} + \sum_{h=1}^{N_m} c_e[h] \cdot E[h] + \sum_{\rho=1}^{N_{TOU}} c_d[\rho] \cdot \max_{j \in \mathcal{H}_\rho} \{P[j]\} \quad (1)$$

where C_{fix} is an infrastructure charge ($), $E[h]$ is the energy consumed (kWh) by the building in period h, $P[j]$ is the power demand (kW) of the building in period j, N_m is the total number of time periods in the month, N_{TOU} is the number of TOU demand charge periods, $c_e[h]$ is the cost of energy ($/kWh) at time h, $c_d[\rho]$ is the cost of demand ($/kW) for the TOU period ρ, and $\mathcal{H}_\rho$ represents the periods in the month when the TOU demand charge ρ is active.

This paper investigates the effects of different economic MPC methods optimizing Eq. (1), looking at both the building owner and the grid utility perspectives, with a focus on commercial buildings. Most of the MPC works encountered in the literature focus on the energy component of Eq. (1) [3]. When peak demand charges are tackled, the approach is generally a "best-effort" manner, for it strives to reduce the peak as much as possible at every control step.

In [5]–[7], MPC formulations include a maximum demand penalty alongside the incremental energy consumption. The coefficient of the demand component in the objective function is either set to a single monthly cost of demand or a weight whose tuning is not discussed in detail. The authors in [8] present a stochastic optimization method that keeps the grid power purchased below a defined demand charge threshold, considering multi-peak periods. However, these aforementioned methods do not consider that the utility demand bill is based on the highest peak of the entire month, in each TOU period (if there are multiple). Given a reasonable optimization horizon on the order of 6-24 hours, rather than the whole month, it is therefore, incorrect to weigh the monthly demand cost against only a day's worth of energy cost in the objective. This will bias the optimization to reduce demand as much as possible each day, at the expense of a higher energy cost, especially if a high peak demand has already been set earlier.

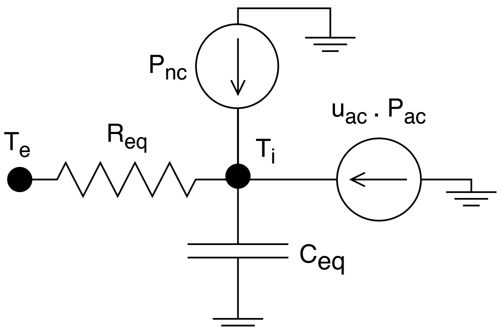

Figure 1: Building RC thermal model

Therefore, this study also introduces an *incremental* approach for optimizing energy and demand costs with MPC, which better accounts for the trade-off between energy and demand costs during an optimization horizon, and considers the general case of multiple TOU demand charge periods. Case studies on a large range of commercial building scenarios are carried out to highlight advantages and disadvantages of each method.

II. System modelling and control

A. Commercial building model

The commercial building power consumption is modeled, at each time instant *t*, as the combination of uncontrollable demand $P_{nc}(t)$ [kW] and flexible power demand of the Air Conditioner (AC) unit $u_{AC}(t)$:

$$x(t) = P_{nc}(t) + u_{AC}(t) \cdot \frac{P_{AC}^{cap}}{k_{cop}} \tag{2}$$

where P_{AC}^{cap} [kW] is the maximum thermal power capacity of the AC unit (negative value), and k_{cop} is the average Coefficient Of Performance (COP) of the AC unit, considered to be independent of ambient conditions. It is therefore assumed that only the cooling electricity demand can be controlled. Furthermore, the AC system is allowed to be continuously controlled:

$$0 \leq u_{AC}(t) \leq 1 \tag{3}$$

The AC unit cools down the whole building, represented as a single zone whose temperature evolves according to the following equation [9] (see Fig. 1):

$$C_{eq} \cdot \dot{T}_i(t) = u_{AC}(t) \cdot P_{AC}^{cap} + P_{nc}(t) + \frac{1}{R_{eq}} \cdot [T_i(t) - T_e(t)] \tag{4}$$

where $T_i(t)$ and $T_e(t)$ are the internal and external temperature at time t respectively and C_{eq} and R_{eq} are the equivalent capacitance and resistance of the thermal zone model respectively. In this model, the uncontrollable loads dissipate electricity entirely as heat into the zone. Moreover, the sun and human heat gain have not been modelled, for sake of simplicity.

In order to model the thermal storage of the building internal mass, a zone capacitance multiplier is applied to the air capacitance [10]:

$$C_{eq} = k_{mass} \cdot C_{air} \tag{5}$$

The thermal comfort of the commercial building occupants must be ensured at any time t:

$$T_i^{min}(t) \leq T_i(t) \leq T_i^{max}(t) \tag{6}$$

The chosen model simplifies drastically the behavior of real commercial buildings, and would therefore never be used to simulate a specific building accurately. However, rather than modelling a specific commercial building very accurately, this study intends to assess the grid-level impact of MPC control methods. The model is therefore suitable enough for this purpose, as it represents a generic commercial building, containing both uncontrollable load and flexible demand.

B. MPC formulations

The building energy is managed by the MPC controller that sets the AC power demand every 15 minutes. The aforementioned building model is directly used to generate each new set point, along with weather and load forecast, through the solving of an optimization problem over a finite horizon H. The traditional commercial economic MPC found in the literature can be formulated as follow [3], [5]–[7]:

$$\begin{aligned} \underset{x}{\text{minimize}} \; & \sum_{h=0}^{H-1} c_e[h] \cdot x[h] \cdot dt + k_d \cdot z \\ \text{s.t.} \; & \text{discretized Equations (2) and (4)} \\ & \text{constraints (3) and (6)} \\ & z \geq 0 \\ & z \geq x[h] \quad \forall h = 0, \ldots, H-1 \end{aligned} \tag{7}$$

where z is a slack variable used to model the maximum demand in the horizon. In the common case where $k_d = 0$, the MPC only reacts to the energy component of the bill.

However, this formulation does not take into account the possible multiple TOU demand charges that can occur for some commercial tariffs. Moreover, it always tries to minimize the peak demand in a short horizon H, without taking into account the last peak it has already set during the month. The above MPC formulation, therefore, overestimates the weight of the demand cost in the horizon, particularly later in the month.

This study therefore proposes a new formulation that includes an *incremental* form of the demand cost and that takes into account multiple demand cost periods:

$$\begin{aligned} \underset{x}{\text{minimize}} \; & \sum_{h=0}^{H-1} c_e[h] \cdot x[h] \cdot dt \quad + \sum_{\rho=1}^{N_{TOU}} c_d[\rho] \cdot z_\rho \\ \text{s.t.} \; & \text{discretized Equations (2) and (4)} \\ & \text{constraints (3) and (6)} \\ & z_\rho \geq 0 \qquad \forall \rho = 0, \ldots, N_{TOU} \\ & z_\rho \geq x[h] - x_\rho^{thr} \qquad \forall h \in \mathcal{H}_\rho \cap \{0, \ldots, H-1\} \end{aligned} \tag{8}$$

Table I: Description of the implemented MPC methods.

Objective \ Scenarios	A	B	C.1	C.2
Energy	X	X	X	X
Peak best effort		X		
Incremental TOU Multi-Peak			X	X
Max demand prediction				X

In this MPC formulation, z_ρ is a slack variable representing the maximum demand in the corresponding TOU periods ρ of the horizon H, and x_ρ^{thr} is a peak demand threshold that only penalizes demand cost if the horizon peak is larger than this threshold. It could be set to the maximum peak encountered since the beginning of the billing period, or to a prediction of what will be the maximum demand during the month. Therefore, $x_\rho^{thr} = max(x_\rho^{seen}, x_\rho^{exp})$ is the maximum of x_ρ^{seen}, the already seen maximum demand in $\mathcal{H}_\rho$, and x_ρ^{exp}, the expected maximum demand in the billing period in $\mathcal{H}_\rho$. We term this formulation as *incremental* because the objective function represents the incremental portion of the monthly bill for the given time horizon, in both energy and demand costs.

III. Case study and discussion

This section presents a case study that highlights the impacts of various MPC approaches on building owner bill, shifting load potential and peak demand. The study consists in multiple monthly simulations of commercial buildings energy consumption. It intends to provide a qualitative analysis of the grid-level effects induced by each of the control strategies.

Table I describes the MPC objectives and features implemented in this case study. The first one, "*A. Energy only*", is the most encountered in the literature and only optimizes on the energy part of the bill, hence setting $k_d = 0$ in Eq.(7). The second one, "*B. Peak best effort*", strives to reduce the peak demand in the MPC horizon, setting k_d to the total demand cost in Eq.(7). The method "*C1. Incremental TOU Multi-Peak*" implements the optimization as in Eq.(8), setting x_ρ^{thr} to the maximum peak already encountered earlier in the month and taking into account the multiple TOU demand charges. Compared to the former, the method "*C2. Incremental TOU Multi-Peak with prediction*" benefits from the knowledge of the maximum demand that will occur during the month, stored in x_ρ^{thr}. Methods such as the one described in [11] could be used for estimating this peak. For practical purpose of these simulations, this peak forecast is retrieved from the simulation results of method B.

Each MPC method is evaluated based the following metrics:

- **Maximum peak demand**: the maximum power demand (kW) throughout the month, averaged over 15 minutes.
- **Monthly bill**: the bill ($) paid by the building owner at the end of the month, according to Eq.(1).
- **Load shifting capacity**: the energy (kWh) consumed during CPP DR events, initiated by the utility.

Table II: PG&E summer tariff rates - off-peak = week-days 10pm - 8am, week-ends & holidays, mid-peak = week-days 8am-12pm & 6pm-10pm, on-peak = week-days 12pm-6pm.

	Energy ($/kWh)		Demand ($/kW)	
Time periods	A-10	E-19	A-10	E-19
off-peak	0.134	0.085	0	0
mid-peak	0.163	0.111	0	5.18
on-peak	0.218	0.152	0	18.64
All time	/	/	18.26	17.57
CPP increment	0.9	1.2	0	0

Table III: Simulated buildings equivalent parameters

Parameter & data	Retail Store	Secondary School
R_{eq} [K/W]	$4.311e^{-4}$	$4.774e^{-5}$
C_{eq} [J/K]	$1.4e^{7}$	$1.5e^{8}$
k_{mass}	4 / 8	3 / 6
P_{AC}^{cap} [kW]	-94.5 / -154	-595 / -735
k_{cop}	3.5	3.5
$P_{nc}(t)$ [kW]	See Fig. 2 (left)	See Fig. 2 (right)
Tariff	A-10	E-19
$T_i^{min}(t)$ [°C]	21 from 6am to 9pm, 16 else	
$T_i^{max}(t)$ [°C]	24 from 6am to 9pm, 30 else	
$T_e(t)$ [°C]	See Fig. 3	
CPP days	8th, 17th, and 27th of July	

This study considers two different TOU commercial tariffs of Pacific Gas & Electricity (PG&E), an electrical utility in California (see Table II). Both tariffs have TOU energy rates and a demand charge applied to all time periods of the month. In addition, tariff E-19 includes TOU demand charges, increasing peak demand prices during mid-peak and on-peak hours. On top of these basic rates, PG&E can also trigger CPP events from 2pm to 6pm.

Due to the simplicity of the model used for the simulations, the results in this section represent the best-case scenario that could be encountered in real-life applications. A major hypothesis is to consider the same model for the MPC and the simulated building. Though unrealistic, this is useful in this case to mitigate the inherent inaccuracy of the control model.

A. Building parameters and simulation data

The DOE Commercial Building Dataset [12] is used to derive commercial buildings simulation parameters (Eq. (2), (4)). This dataset gathers generic building information (*e.g.* thermal envelope, zones size, uncontrollable load magnitude) and schedules (*e.g.* occupancy, heating/cooling, internal gain) for a wide range of commercial buildings. Two distinct buildings have been chosen for this study (see Table III):

- Retail store: a medium-size commercial building, made of few conditioned zones on a single level. Its internal mass coefficient can either be 4 (lightweight) or 8

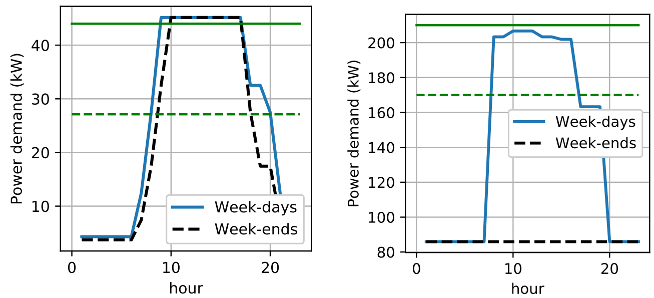

Figure 2: Uncontrollable load profile: (left) Retail store (right) Secondary school. Maximum AC capacity in green: (filled) Warm and Hot environment, (dashed) Mild environment.

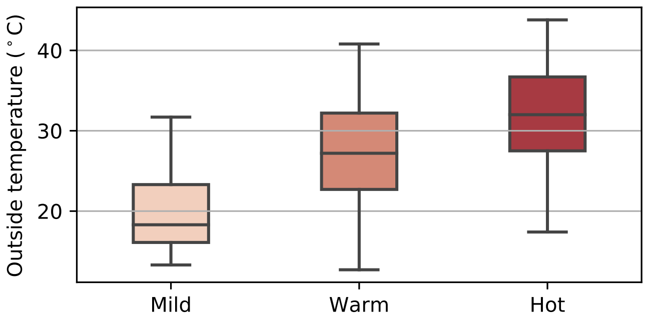

Figure 3: Outside temperatures in July, for 3 environments.

(heavyweight). The aggregated uncontrollable load signal is shown in Fig. 2 (left).

- Secondary school: a large commercial building, made of > 40 conditioned zones, spread on two levels. Its internal mass coefficient can either be 3 (lightweight) or 6 (heavyweight). The aggregated uncontrollable load signal is shown in Fig. 2 (right).

Considering a single conditioned zone with a continuous AC system is a simplification from reality, as in practice, individual zone AC units would work asynchronously or be part of a larger multizone AC system. Nevertheless, this leads to results that are independent from a specific configuration and indicative of the performance of each MPC formulation. The system COP averages the group of individual units.

The outside temperature data depends on the climate zone (Mild, Warm, or Hot) and represents a typical month in the summer in US [13]. Fig. 3 statistically describes the hourly signals used for the simulations. Depending on the simulated climate, the AC thermal capacity in each building can take a different value. In order to assess the shifting capability under realistic DR events, three CPP events increase the prices of energy for the three hottest days (see Table III).

Applying the various MPC methods to simulations of different buildings and environments allows for a sensitivity analysis of the results, instead of focusing on a specific configuration. In this study, the MPC methods of Table I are evaluated on

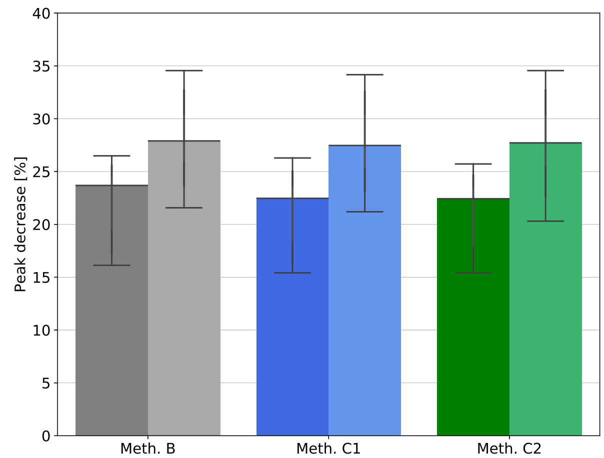

Figure 4: Relative *decrease* of the maximum power demand of the month, compared to control Method A. For each control method: (left bar) Retail store, (right bar) Secondary school.

all possible configuration triplets {blg type, internal mass, climate}. This corresponds to a total of 12 simulations per MPC method. Every simulation spans over an entire month, with an MPC update every 15 minutes. At each time step, the internal temperature is kept within bounds, while optimizing the objective over a time horizon. The MPC horizon is set to 12 hours, large enough to foresee the price, temperature, and uncontrollable power variation. From an implementation standpoint, the Python package *cvxpy* wraps the optimization formulations, and calls the open-source ECOS solver [14]; the package *control* discretizes the continuous thermal model (Eq.(4)), for both the simulation and the MPC model.

B. Results and discussion

Given the difference in the orders of magnitude between the two buildings electricity demand, a relative comparison of the results makes more sense. The method A "Energy Only" will serve as the baseline, for it is the most encountered in the literature. The results therefore present the increase/decrease of the aforementioned metrics by the three other MPC control methods relative to the "A. Energy Only" formulation.

Fig. 4 shows the relative decrease of the **maximum peak demand** for the control methods B, C1, & C2 with respect to the control method A. One observes a tremendous decrease of the peak demand, ranging from 15% to 35%, due to the fact that the method A does not take the demand cost into account. The secondary school displays a larger gain than the retail store. The explanation is twofold. First, the proportion of controllable to uncontrollable load is higher for the secondary school, on average over time. Second, the tariff E-19, which the secondary school falls under, penalizes demand more than the A-10. As for the peak performance, the method B reduces peak demand slightly more than the methods C1&C2 for all of the cases. This is due to the method B using the full demand cost as the weight to penalize the demand over the MPC

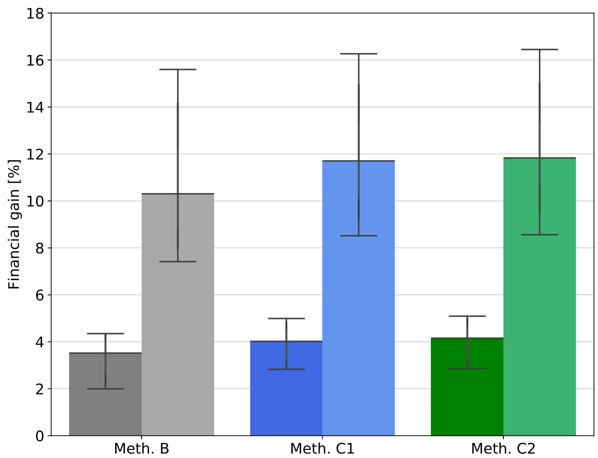

Figure 5: Relative *decrease* of the monthly bill, compared to control Method A. For each control method: (left bar) Retail store, (right bar) Secondary school.

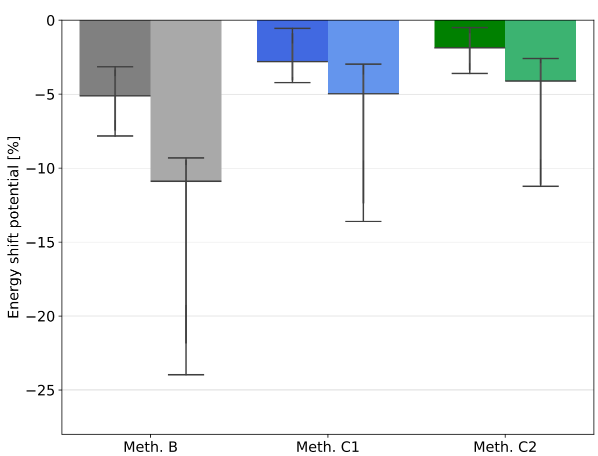

Figure 6: Relative *loss* of energy shifting potential during CPP events, compared to control Method A. For each control method: (left bar) Retail store, (right bar) Secondary school.

horizon. Nevertheless, the incremental methods C1&C2 only worsen the peak by 2%, in the worst case.

Fig. 5 shows the relative decrease of the **monthly bill** for the control methods B, C1, & C2 with respect to the control method A. The large gap between the retail store and the secondary school is mainly driven by the lower peak to average ratio of the latter, strongly penalized by the tariff E-19 with higher demand charges. Methods C1 & C2 show an improvement of about 1-2% compared to the method B. The improvement can be explained by a better management of the TOU demand charges, whereas the method B does not differentiate the periods of the day. Moreover, the incremental demand charge feature of C1 & C2 enables them to dictate load shifting with energy rate fluctuations. The knowledge of the maximum peak in the month with method C2 slightly improves the bill compared to method C1. This feature leads to better pre-cooling decisions during the beginning of the month, where method C1 does not risk setting a new peak. While the bill gain difference seems low, it is to be compared with an already-optimized system,

Fig. 6 shows the relative decrease (negative increase) of the **load shifting capacity** for the control methods B, C1, & C2 with respect to the control method A. This metric is computed by summing the energy consumption during the three CPP events of the month. Method A is therefore the best, as it only optimizes on energy cost and can shift the demand as much as possible. Compared to this baseline, method B clearly lacks the ability to shift load, especially under a tariff that strongly penalizes the peak demand (secondary school). Incremental methods C1 & C2 are capable of cutting the loss of load shifting potential in half compared to method B. This effect would even be more marked as the proportion of controllable load is increased. This is due to the fact that these methods leverage the knowledge of past behavior and prediction of future behavior, specifically being able to trade off cost savings from load shifting to cost increases from setting a new demand peak. The prediction of monthly maximum peak demand improves the shifting potential by about 1-2%.

Analyzing the details of timeseries power consumption results allows for further understanding of metric trade-offs and global trend of each method. Fig. 7 (left) plots the hourly power consumption induced by each control method, averaged over all of the simulations of the secondary school except the CPP days. The four methods exhibit the same behavior in the early morning (until 6am) and end of the day (from 5pm), due to the low risk of setting a new monthly peak. As the first TOU energy rate increment appears, they all pre-cool the zone. However, subsequent behavior diverges. Method A reduces the energy as much as possible, whereas incremental methods C1&C2 only slightly reduce it. Method B keeps a constant increase in power demand, disregarding the energy rate increase. At the hours before the on-peak period, the method A fully pre-cools the building to the lower bound of temperature to allow maximum free-float during the subsequent period of higher energy price. Methods C1&C2 enable some pre-cooling, though better account for the tradeoff between energy shifting and setting a new peak demand. The monthly peak demand prediction in method C2 allows it to provide slightly more energy shift, as it knows the monthly peak will be set later anyway. Method B does not pre-cool at all since the demand cost prevails on the energy cost.

Fig. 7 (right) presents the last CPP day of the month, for a simulation of the secondary school (warm weather, large internal mass). On this day, the hot temperature prevents methods C1 and C2 from pre-cooling too much without setting a new costly demand peak. Nevertheless, more pre-cooling during the morning than method B still allows for more load reduction during CPP hours, though less than method A.

IV. Conclusion

This study compares existing economic MPC formulations applied to commercial buildings and introduces a new *incre-*

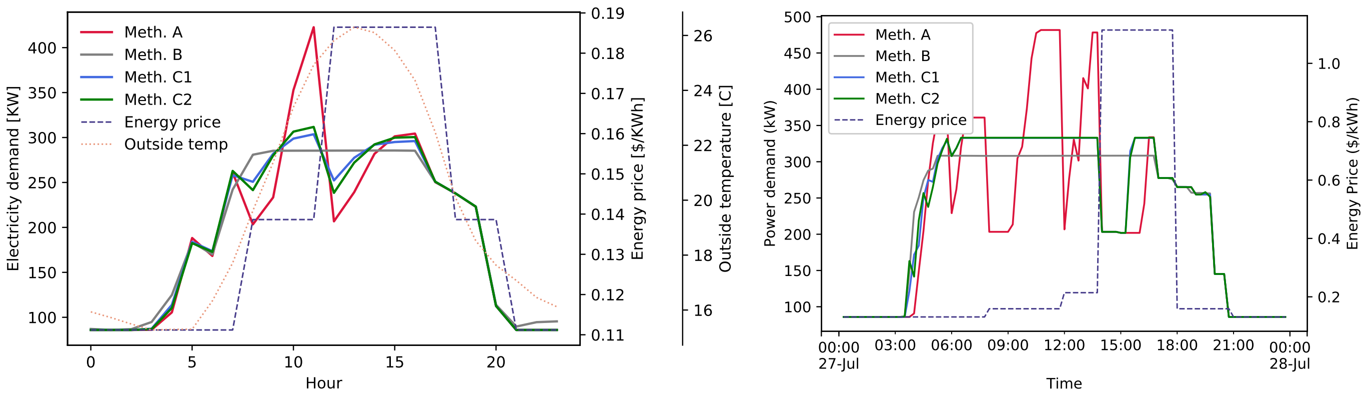

Figure 7: Timeseries analysis of "secondary school" control methods: (left) hourly mean of power consumption, energy price and outside temperature throughout all simulations (right) 15-min power consumption on the 3rd CPP event.

mental method that leverages past behavior of the building. A large number of simulations were carried out on simplified commercial building models, with different climates. Results showed that the traditional MPC method of taking into account demand charges reduces both the peak demand and the electricity bill relative to the solution optimizing for energy costs only. However, it prevents the building from shifting load when needed, such as during CPP events. The incremental formulation improved the building responsiveness to price-based DR signals, while similarly keeping constant or slightly reducing the owner bill and maximum peak demand.

The simulation results highlight that multiple MPC formulations can offer the same value for the user (in terms of utility bill cost) but different grid service capabilities. Ongoing world decarbonisation efforts increasingly encourage the deployment of DR programs to incentivize load shifting and peak load reduction. The tariff structures should, therefore, be carefully designed to optimally leverage building load flexibility offered by different MPC formulations.

Acknowledgment

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