The International Association for the Properties of Water and Steam

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Supplementary Release on Backward Equations for the Functions *T*(*p*,*h*), *v*(*p*,*h*) and *T*(*p*,*s*), *v*(*p*,*s*) for Region 3 of the IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam

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This release contains 18 pages, including this cover page.

This supplementary release has been authorized by the International Association for the Properties of Water and Steam (IAPWS) at its meeting in Vejle, Denmark, 24-29 August, 2003, for issue by its Secretariat. The members of IAPWS are: Argentina and Brazil, Britain and Ireland, Canada, the Czech Republic, Denmark, France, Germany, Italy, Japan, Russia, the United States of America, and associate member Greece.

The backward equations for temperature and specific volume as functions of pressure and enthalpy $T(p,h)$, $v(p,h)$ and as functions of pressure and entropy $T(p,s)$, $v(p,s)$ for region 3 provided in this release are recommended as a supplement to "IAPWS Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" (IAPWS-IF97) [1, 2] and to "Backward Equations for Pressure as a Function of Enthalpy and Entropy $p(h,s)$ to the Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" (referred to here as IAPWS-IF97-S01) [3, 4]. Further details concerning the equations can be found in the corresponding article by H.-J. Kretzschmar et al. [5].

Further information concerning this supplementary release, IAPWS-IF97, IAPWS-IF97-S01, and other releases issued by IAPWS can be obtained from the Executive Secretary of IAPWS or from http://www.iapws.org.

Contents

1 Nomenclature

Thermodynamic quantities:

f Specific Helmholtz free energy
h Specific enthalpy
p Pressure
s Specific entropy
T Absolute temperature [a]
v Specific volume
Δ Difference in any quantity
h Reduced enthalpy, $h = h/h^*$
q Reduced temperature $q = T/T^*$
p Reduced pressure, $p = p/p^*$
r Density
s Reduced entropy, $s = s/s^*$
w Reduced volume, $w = v/v^*$

Root-mean-square value:

$$\Delta x_{\mathrm{RMS}} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\Delta x_n)^2}$$

where Δx_n can be either absolute or percentage difference between the corresponding quantities x; N is the number of Δx_n values (100 million points uniformly distributed over the range of validity in the p-T plane).

Superscripts:

01 Equation of IAPWS-IF97-S01
97 Quantity or equation of IAPWS-IF97
* Reducing quantity
' Saturated liquid state
" Saturated vapor state

Subscripts:

1 Region 1
2 Region 2
3 Region 3
3a Subregion 3a
3b Subregion 3b
3ab Boundary between subregions 3a and 3b
4 Region 4
5 Region 5
B23 Boundary between regions 2 and 3
c Critical point
it Iterated quantity
max Maximum value of a quantity
RMS Root-mean-square value of a quantity
sat Saturation state
tol Tolerated value of a quantity

[a] Note: T denotes absolute temperature on the International Temperature Scale of 1990 (ITS-90).

2 Background

The Industrial Formulation IAPWS-IF97 for the thermodynamic properties of water and steam [1, 2] contains basic equations, saturation equations and equations for the most often used backward functions $T(p,h)$ and $T(p,s)$ valid in the liquid region 1 and the vapor region 2; see Figure 1. IAPWS-IF97 was supplemented by "Backward Equations for Pressure as a Function of Enthalpy and Entropy $p(h,s)$ to the Industrial Formulation 1997 for the Thermodynamic Properties of Water and Steam" [3, 4], which we will refer to as IAPWS-IF97-S01, including equations for the backward function $p(h,s)$ valid in region 1 and region 2.

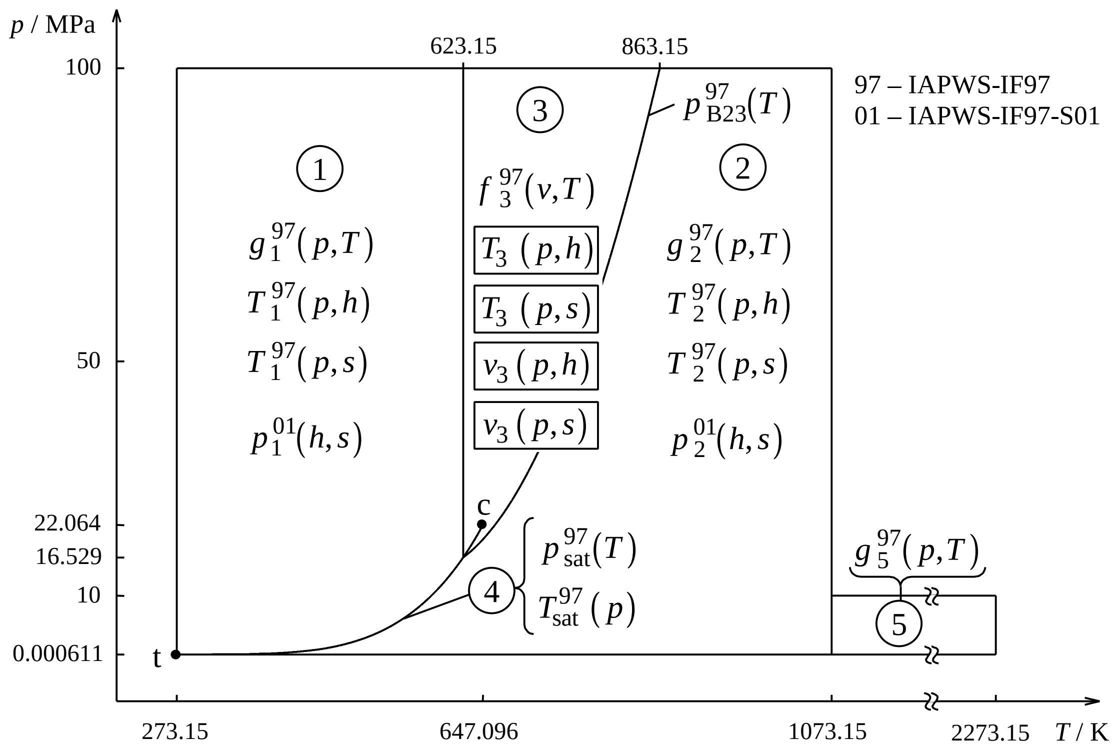

Figure 1. Regions and equations of IAPWS-IF97, IAPWS-IF97-S01, and the backward equations $T(p,h)$, $v(p,h)$, and $T(p,s)$, $v(p,s)$ of this release

In modeling steam power cycles, thermodynamic properties as functions of the variables (p,h) or (p,s) are also required in region 3. It is difficult to perform these calculations with IAPWS-IF97, because they require two-dimensional iterations using the functions $p(v,T)$, $h(v,T)$ or $p(v,T)$, $s(v,T)$ that can be explicitly calculated from the fundamental region 3 equation $f(v,T)$. While these calculations are not frequently required in region 3, the relatively large computing time required for two-dimensional iteration can be significant in process modeling.

In order to avoid such iterations, this release provides equations for the backward functions $T_3(p,h)$, $v_3(p,h)$ and $T_3(p,s)$, $v_3(p,s)$, see Figure 1. With temperature and specific volume calculated from the backward equations, the other properties in region 3 can be calculated using the IAPWS-IF97 basic equation $f_3^{97}(v,T)$.

The numerical consistency with the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ of T and v calculated from these backward equations is sufficient for most applications in heat cycle and steam turbine calculations. For applications where the demands on numerical consistency are extremely high, iterations using the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ may be necessary. In these cases, the equations $T_3(p,h)$, $v_3(p,h)$ and $T_3(p,s)$, $v_3(p,s)$ can be used for calculating very accurate starting values.

The backward equations $T_3(p,h)$, $v_3(p,h)$ and $T_3(p,s)$, $v_3(p,s)$ can only be used in their ranges of validity described in Section 4. They should not be used for determining any thermodynamic derivatives.

In any case, depending on the application, a conscious decision is required whether to use the backward equations $T_3(p,h)$, $v_3(p,h)$ and $T_3(p,s)$, $v_3(p,s)$ or to calculate the corresponding values by iterations from the basic equation of IAPWS-IF97.

3 Numerical Consistency Requirements

The permissible value for the numerical consistency $|\Delta T|_{\mathrm{tol}} = 25\ \mathrm{mK}$ of the backward functions $T_3(p,h)$ and $T_3(p,s)$ with the basic equation $f_3^{97}(v,T)$ was determined by IAPWS [6, 7] as a result of an international survey.

The permissible value Δv_{tol} for the numerical consistency for the equations $v_3(p,h)$ and $v_3(p,s)$ can be estimated from the total differentials

$$\Delta v_{\mathrm{tol}} = \left(\frac{\partial v}{\partial T}\right)_h \Delta T_{\mathrm{tol}} + \left(\frac{\partial v}{\partial h}\right)_T \Delta h_{\mathrm{tol}} \quad \text{and} \quad \Delta v_{\mathrm{tol}} = \left(\frac{\partial v}{\partial T}\right)_s \Delta T_{\mathrm{tol}} + \left(\frac{\partial v}{\partial s}\right)_T \Delta s_{\mathrm{tol}} \ ,$$

where $\left(\frac{\partial v}{\partial T}\right)_h$, $\left(\frac{\partial v}{\partial h}\right)_T$, $\left(\frac{\partial v}{\partial T}\right)_s$, and $\left(\frac{\partial v}{\partial s}\right)_T$ are derivatives [8] calculated from the IAPWS-IF97 basic equation and Δh_{tol} and Δs_{tol} are values determined by IAPWS for the adjacent region 1 and subregion 2c [9], see Table 1. The resulting permissible specific volume difference is $|\Delta v/v|_{\mathrm{tol}} = 0.01\ \%$ for both functions $v_3(p,h)$ and $v_3(p,s)$.

At the critical point $\left[T_{\mathrm{c}} = 647.096\mathrm{K},\ v_{\mathrm{c}} = 1/\left(322\ \mathrm{kg\ m^{-3}}\right)\right]$, more stringent consistency requirements were arbitrarily set. These were $|\Delta T|_{\mathrm{tol}} = 0.49\ \mathrm{mK}$ and $|\Delta v/v|_{\mathrm{tol}} = 0.0001\ \%$.

Table 1. Numerical consistency values $|\Delta T|_{\mathrm{tol}}$ of [6] required for $T_3(p,h)$ and $T_3(p,s)$, values $|\Delta h|_{\mathrm{tol}}$, $|\Delta s|_{\mathrm{tol}}$ of [9], and resulting tolerances $|\Delta v/v|_{\mathrm{tol}}$ required for $v_3(p,h)$ and $v_3(p,s)$

	$\lvert\Delta T\rvert_{\mathrm{tol}}$	$\lvert\Delta h\rvert_{\mathrm{tol}}$	$\lvert\Delta s\rvert_{\mathrm{tol}}$	$\lvert\Delta v/v\rvert_{\mathrm{tol}}$
Region 3	25 mK	80 $\mathrm{J\,kg^{-1}}$	0.1 $\mathrm{J\,kg^{-1}\,K^{-1}}$	0.01 %
Critical Point	0.49 mK	-	-	0.0001 %

4 Structure of the Equation Set

The equation set consists of backward equations $T(p,h)$, $v(p,h)$ and $T(p,s)$, $v(p,s)$ for region 3.

Region 3 is defined by:

$$623.15\ \mathrm{K} \le T \le 863.15\ \mathrm{K} \quad \text{and} \quad p_{\mathrm{B23}}^{97}(T) \le p \le 100\ \mathrm{MPa}\,,$$

where p_{B23}^{97} represents the B23 equation of IAPWS-IF97. Figure 2 shows the way in which region 3 is divided into the two subregions 3a and 3b.

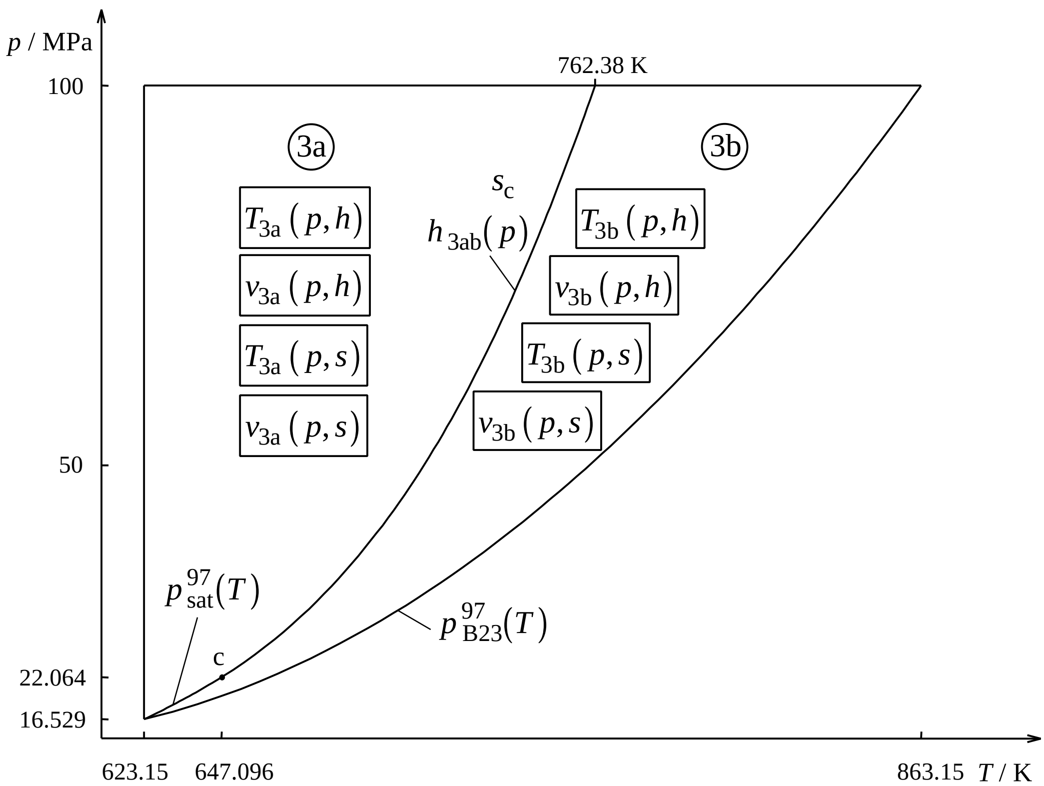

Figure 2. Division of region 3 into two subregions 3a and 3b for the backward equations $T(p,h)$, $v(p,h)$ and $T(p,s)$, $v(p,s)$

Table 2 shows the decisions which have to be made in order to find the correct subregion for the functions $T(p,h)$, $v(p,h)$ and $T(p,s)$, $v(p,s)$.

Table 2. Criteria for finding the correct subregion, 3a or 3b, for the backward functions $T(p,h)$, $v(p,h)$ and $T(p,s)$, $v(p,s)$

Backward Functions $T(p,h)$, $v(p,h)$			Backward Functions $T(p,s)$, $v(p,s)$		
	Subregion			Subregion	
	3a	3b		3a	3b
for $p < p_c$:	$h \le h'(p)$	$h \ge h''(p)$	for $p < p_c$:	$s \le s'(p)$	$s \ge s''(p)$
for $p \ge p_c$:	$h \le h_{3ab}(p)$	$h > h_{3ab}(p)$	for $p \ge p_c$:	$s \le s_c$	$s > s_c$

For pressures less than the critical pressure $p_c = 22.064\ \mathrm{MPa}$, the saturation line is the boundary between subregions 3a and 3b. That means for the functions $T(p,h)$ and $v(p,h)$, if the given specific enthalpy h is less than or equal to $h'(p)$ calculated from the given pressure p on the saturated liquid line, then the point of state to be calculated is located in subregion 3a. If the given enthalpy h is greater than or equal to $h''(p)$ calculated on the saturated vapor line, then the point of state is located in subregion 3b. Otherwise, the point is in the two-phase region. In that case, the saturation temperature equation $T_{sat}^{97}(p)$ and the basic equation $f_3^{97}(v,T)$ of IAPWS-IF97 can be used to calculate the temperature and the specific volume from the given pressure and the given enthalpy. The decisions are analogous for the functions $T(p,s)$ and $v(p,s)$.

For pressures greater than or equal to p_c, the boundary between the subregions 3a and 3b corresponds to the critical isentropic line $s = s_c$, see Figure 2. For the functions $T(p,s)$ and $v(p,s)$, input points can be tested directly to identify the subregion since the specific entropy is an independent variable. If the given specific entropy s is less than or equal to

$$s_c = 4.412\,021\,482\,234\,76\ \mathrm{kJ\ kg^{-1}\ K^{-1}},$$

then the state point to be calculated is located in subregion 3a; otherwise it is in subregion 3b. In order to decide which $T(p,h)$, $v(p,h)$ equation, 3a or 3b, must be used for given values of p and h, the boundary equation $h_{3ab}(p)$, Eq. (1), has to be used, see Figure 2. This equation is a polynomial of the third degree and reads

$$\frac{h_{3ab}(p)}{h^*} = \eta(\pi) = n_1 + n_2\pi + n_3\pi^2 + n_4\pi^3\ , \tag{1}$$

where $\eta = h/h^*$ and $\pi = p/p^*$ with $h^* = 1\ \mathrm{kJ\ kg^{-1}}$ and $p^* = 1\mathrm{MPa}$. The coefficients n_1 to n_4 of Eq. (1) are listed in Table 3. The range of the equation $h_{3ab}(p)$ is from the critical point to 100 MPa. The related temperature at 100 MPa is $T = 762.380\,873\,481$ K. Equation (1) does not exactly describe the critical isentropic line. The maximum specific entropy deviation was determined as

$$\left|\Delta s_{3\text{ab}}\right|_{\max} = \left| s_3^{97}\left(T_{\text{it}}^{97}\,(p, h_{3\text{ab}}(p)\,), v_{\text{it}}^{97}(\,p, h_{3\text{ab}}(p)\,)\,\right) - s_{\text{c}} \right|_{\max} = 0.66\ \text{J kg}^{-1}\text{K}^{-1}\ ,$$

where T_{it}^{97} and v_{it}^{97} were obtained by iterations using the derivatives $p_3^{97}(v,T)$ and $s_3^{97}(v,T)$ of the IAPWS-IF97 basic equation for region 3.

Table 3. Numerical values of the coefficients of the equation $h_{3\text{ab}}(p)$ in its dimensionless form, Eq. (1), for defining the boundary between subregions 3a and 3b [a]

i	n_i	i	n_i
1	$0.201\,464\,004\,206\,875 \times 10^{4}$	3	$-\,0.219\,921\,901\,054\,187 \times 10^{-1}$
2	$0.374\,696\,550\,136\,983 \times 10^{1}$	4	$0.875\,131\,686\,009\,950 \times 10^{-4}$

[a] It is recommended that programmed functions be verified using 8 byte real values for all variables.

If the given specific enthalpy h is greater than $h_{3\text{ab}}(p)$ calculated from the given pressure p, then the state point to be calculated is located in subregion 3b, otherwise it is in subregion 3a (see Figure 2).

Note, Eq. (1) does not correctly simulate the isentropic line $s = s_{\text{c}}$ at pressures lower than p_{c}. However, the calculated values $h_{3\text{ab}}(p)$ are not higher than $h''(p)$ and not lower than $h'(p)$.

For *computer-program verification*, Eq. (1) gives the following *p*-*h* point:

$$p = 25\ \text{MPa}\ ,\ h_{3\text{ab}}(p) = 2.095\,936\,454 \times 10^{3}\ \text{kJ kg}^{-1}.$$

5 Backward Equations $T(p,h)$ and $v(p,h)$ for Subregions 3a and 3b

5.1 The Equations $T(p,h)$

The backward equation $T_{3\text{a}}(p,h)$ for subregion 3a has the following dimensionless form:

$$\frac{T_{3\text{a}}(p,h)}{T^*} = \theta_{3\text{a}}(\pi,\eta) = \sum_{i=1}^{31} n_i\,(\pi + 0.240)^{I_i}\,(\eta - 0.615)^{J_i}\ , \tag{2}$$

where $\theta = T/T^*$, $\pi = p/p^*$, and $\eta = h/h^*$ with $T^* = 760\ \text{K}$, $p^* = 100\ \text{MPa}$, and $h^* = 2300\ \text{kJ kg}^{-1}$. The coefficients n_i and exponents I_i and J_i of Eq. (2) are listed in Table 4.

The backward equation $T_{3\text{b}}(p,h)$ for subregion 3b reads in its dimensionless form

$$\frac{T_{3\text{b}}(p,h)}{T^*} = \theta_{3\text{b}}(\pi,\eta) = \sum_{i=1}^{33} n_i\,(\pi + 0.298)^{I_i}\,(\eta - 0.720)^{J_i}\ , \tag{3}$$

where $q = T/T^*$, $p = p/p^*$, and $h = h/h^*$ with $T^* = 860\ \mathrm{K}$, $p^* = 100\ \mathrm{MPa}$, and $h^* = 2800\ \mathrm{kJ\ kg^{-1}}$. The coefficients n_i and exponents I_i and J_i of Eq. (3) are listed in Table 5.

Computer-program verification

To assist the user in computer-program verification of Eqs. (2) and (3), Table 6 contains test values for calculated temperatures.

Table 4. Coefficients and exponents of the backward equation $T_{3a}(p,h)$ for subregion 3a in its dimensionless form, Eq. (2)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	0	$-0.133\,645\,667\,811\,215 \times 10^{-6}$	17	−3	0	$-0.384\,460\,997\,596\,657 \times 10^{-5}$
2	−12	1	$0.455\,912\,656\,802\,978 \times 10^{-5}$	18	−2	1	$0.337\,423\,807\,911\,655 \times 10^{-2}$
3	−12	2	$-0.146\,294\,640\,700\,979 \times 10^{-4}$	19	−2	3	$-0.551\,624\,873\,066\,791$
4	−12	6	$0.639\,341\,312\,970\,080 \times 10^{-2}$	20	−2	4	$0.729\,202\,277\,107\,470$
5	−12	14	$0.372\,783\,927\,268\,847 \times 10^{3}$	21	−1	0	$-0.992\,522\,757\,376\,041 \times 10^{-2}$
6	−12	16	$-0.718\,654\,377\,460\,447 \times 10^{4}$	22	−1	2	$-0.119\,308\,831\,407\,288$
7	−12	20	$0.573\,494\,752\,103\,400 \times 10^{6}$	23	0	0	$0.793\,929\,190\,615\,421$
8	−12	22	$-0.267\,569\,329\,111\,439 \times 10^{7}$	24	0	1	$0.454\,270\,731\,799\,386$
9	−10	1	$-0.334\,066\,283\,302\,614 \times 10^{-4}$	25	1	1	$0.209\,998\,591\,259\,910$
10	−10	5	$-0.245\,479\,214\,069\,597 \times 10^{-1}$	26	3	0	$-0.642\,109\,823\,904\,738 \times 10^{-2}$
11	−10	12	$0.478\,087\,847\,764\,996 \times 10^{2}$	27	3	1	$-0.235\,155\,868\,604\,540 \times 10^{-1}$
12	−8	0	$0.764\,664\,131\,818\,904 \times 10^{-5}$	28	4	0	$0.252\,233\,108\,341\,612 \times 10^{-2}$
13	−8	2	$0.128\,350\,627\,676\,972 \times 10^{-2}$	29	4	3	$-0.764\,885\,133\,368\,119 \times 10^{-2}$
14	−8	4	$0.171\,219\,081\,377\,331 \times 10^{-1}$	30	10	4	$0.136\,176\,427\,574\,291 \times 10^{-1}$
15	−8	10	$-0.851\,007\,304\,583\,213 \times 10^{1}$	31	12	5	$-0.133\,027\,883\,575\,669 \times 10^{-1}$
16	−5	2	$-0.136\,513\,461\,629\,781 \times 10^{-1}$				

Table 5. Coefficients and exponents of the backward equation $T_{3b}(p,h)$ for subregion 3b in its dimensionless form, Eq. (3)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	0	$0.323\,254\,573\,644\,920 \times 10^{-4}$	18	−3	5	$-0.307\,622\,221\,350\,501 \times 10^{1}$
2	−12	1	$-0.127\,575\,556\,587\,181 \times 10^{-3}$	19	−2	0	$-0.574\,011\,959\,864\,879 \times 10^{-1}$
3	−10	0	$-0.475\,851\,877\,356\,068 \times 10^{-3}$	20	−2	4	$0.503\,471\,360\,939\,849 \times 10^{1}$
4	−10	1	$0.156\,183\,014\,181\,602 \times 10^{-2}$	21	−1	2	$-0.925\,081\,888\,584\,834$
5	−10	5	$0.105\,724\,860\,113\,781$	22	−1	4	$0.391\,733\,882\,917\,546 \times 10^{1}$
6	−10	10	$-0.858\,514\,221\,132\,534 \times 10^{2}$	23	−1	6	$-0.773\,146\,007\,130\,190 \times 10^{2}$
7	−10	12	$0.724\,140\,095\,480\,911 \times 10^{3}$	24	−1	10	$0.949\,308\,762\,098\,587 \times 10^{4}$
8	−8	0	$0.296\,475\,810\,273\,257 \times 10^{-2}$	25	−1	14	$-0.141\,043\,719\,679\,409 \times 10^{7}$
9	−8	1	$-0.592\,721\,983\,365\,988 \times 10^{-2}$	26	−1	16	$0.849\,166\,230\,819\,026 \times 10^{7}$
10	−8	2	$-0.126\,305\,422\,818\,666 \times 10^{-1}$	27	0	0	$0.861\,095\,729\,446\,704$
11	−8	4	$-0.115\,716\,196\,364\,853$	28	0	2	$0.323\,346\,442\,811\,720$
12	−8	10	$0.849\,000\,969\,739\,595 \times 10^{2}$	29	1	1	$0.873\,281\,936\,020\,439$
13	−6	0	$-0.108\,602\,260\,086\,615 \times 10^{-1}$	30	3	1	$-0.436\,653\,048\,526\,683$
14	−6	1	$0.154\,304\,475\,328\,851 \times 10^{-1}$	31	5	1	$0.286\,596\,714\,529\,479$
15	−6	2	$0.750\,455\,441\,524\,466 \times 10^{-1}$	32	6	1	$-0.131\,778\,331\,276\,228$
16	−4	0	$0.252\,520\,973\,612\,982 \times 10^{-1}$	33	8	1	$0.676\,682\,064\,330\,275 \times 10^{-2}$
17	−4	1	$-0.602\,507\,901\,232\,996 \times 10^{-1}$				

Table 6. Selected temperature values calculated from Eqs. (2) and (3) [a]

Equation	p / MPa	h / kJ kg^{-1}	T / K
$T_{3a}(p,h)$, Eq. (2)	20	1700	$6.293\,083\,892 \times 10^2$
	50	2000	$6.905\,718\,338 \times 10^2$
	100	2100	$7.336\,163\,014 \times 10^2$
$T_{3b}(p,h)$, Eq. (3)	20	2500	$6.418\,418\,053 \times 10^2$
	50	2400	$7.351\,848\,618 \times 10^2$
	100	2700	$8.420\,460\,876 \times 10^2$

[a] It is recommended that programmed functions be verified using 8 byte real values for all variables.

5.2 The Equations $v(p,h)$

The backward equation $v_{3a}(p,h)$ for subregion 3a has the following dimensionless form:

$$\frac{v_{3a}(p,h)}{v^*} = \omega_{3a}(\pi,\eta) = \sum_{i=1}^{32} n_i\,(\pi + 0.128)^{I_i}\,(\eta - 0.727)^{J_i} \quad , \qquad (4)$$

where $\omega = v/v^*$, $\pi = p/p^*$, and $\eta = h/h^*$ with $v^* = 0.0028\ \mathrm{m^3\ kg^{-1}}$, $p^* = 100\ \mathrm{MPa}$, and $h^* = 2100\ \mathrm{kJ\ kg^{-1}}$. The coefficients n_i and exponents I_i and J_i of Eq. (4) are listed in Table 7.

The backward equation $v_{3b}(p,h)$ for subregion 3b reads in its dimensionless form

$$\frac{v_{3b}(p,h)}{v^*} = \omega_{3b}(\pi,\eta) = \sum_{i=1}^{30} n_i\,(\pi + 0.0661)^{I_i}\,(\eta - 0.720)^{J_i} \quad , \qquad (5)$$

where $\omega = v/v^*$, $\pi = p/p^*$, and $\eta = h/h^*$ with $v^* = 0.0088\ \mathrm{m^3\ kg^{-1}}$, $p^* = 100\ \mathrm{MPa}$, and $h^* = 2800\ \mathrm{kJ\ kg^{-1}}$. The coefficients n_i and exponents I_i and J_i of Eq. (5) are listed in Table 8.

Computer-program verification

To assist the user in computer-program verification of Eqs. (4) and (5), Table 9 contains test values for calculated specific volumes.

Table 7. Coefficients and exponents of the backward equation $v_{3a}(p,h)$ for subregion 3a in its dimensionless form, Eq. (4)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	6	$0.529\,944\,062\,966\,028 \times 10^{-2}$	17	−2	16	$0.568\,366\,875\,815\,960 \times 10^{4}$
2	−12	8	$-0.170\,099\,690\,234\,461$	18	−1	0	$0.808\,169\,540\,124\,668 \times 10^{-2}$
3	−12	12	$0.111\,323\,814\,312\,927 \times 10^{2}$	19	−1	1	$0.172\,416\,341\,519\,307$
4	−12	18	$-0.217\,898\,123\,145\,125 \times 10^{4}$	20	−1	2	$0.104\,270\,175\,292\,927 \times 10^{1}$
5	−10	4	$-0.506\,061\,827\,980\,875 \times 10^{-3}$	21	−1	3	$-0.297\,691\,372\,792\,847$
6	−10	7	$0.556\,495\,239\,685\,324$	22	0	0	$0.560\,394\,465\,163\,593$
7	−10	10	$-0.943\,672\,726\,094\,016 \times 10^{1}$	23	0	1	$0.275\,234\,661\,176\,914$
8	−8	5	$-0.297\,856\,807\,561\,527$	24	1	0	$-0.148\,347\,894\,866\,012$
9	−8	12	$0.939\,353\,943\,717\,186 \times 10^{2}$	25	1	1	$-0.651\,142\,513\,478\,515 \times 10^{-1}$
10	−6	3	$0.192\,944\,939\,465\,981 \times 10^{-1}$	26	1	2	$-0.292\,468\,715\,386\,302 \times 10^{1}$
11	−6	4	$0.421\,740\,664\,704\,763$	27	2	0	$0.664\,876\,096\,952\,665 \times 10^{-1}$
12	−6	22	$-0.368\,914\,126\,282\,330 \times 10^{7}$	28	2	2	$0.352\,335\,014\,263\,844 \times 10^{1}$
13	−4	2	$-0.737\,566\,847\,600\,639 \times 10^{-2}$	29	3	0	$-0.146\,340\,792\,313\,332 \times 10^{-1}$
14	−4	3	$-0.354\,753\,242\,424\,366$	30	4	2	$-0.224\,503\,486\,668\,184 \times 10^{1}$
15	−3	7	$-0.199\,768\,169\,338\,727 \times 10^{1}$	31	5	2	$0.110\,533\,464\,706\,142 \times 10^{1}$
16	−2	3	$0.115\,456\,297\,059\,049 \times 10^{1}$	32	8	2	$-0.408\,757\,344\,495\,612 \times 10^{-1}$

Table 8. Coefficients and exponents of the backward equation $v_{3b}(p,h)$ for subregion 3b in its dimensionless form, Eq. (5)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	0	$-0.225\,196\,934\,336\,318 \times 10^{-8}$	16	−4	6	$-0.321\,087\,965\,668\,917 \times 10^{1}$
2	−12	1	$0.140\,674\,363\,313\,486 \times 10^{-7}$	17	−4	10	$0.607\,567\,815\,637\,771 \times 10^{3}$
3	−8	0	$0.233\,784\,085\,280\,560 \times 10^{-5}$	18	−3	0	$0.557\,686\,450\,685\,932 \times 10^{-3}$
4	−8	1	$-0.331\,833\,715\,229\,001 \times 10^{-4}$	19	−3	2	$0.187\,499\,040\,029\,550$
5	−8	3	$0.107\,956\,778\,514\,318 \times 10^{-2}$	20	−2	1	$0.905\,368\,030\,448\,107 \times 10^{-2}$
6	−8	6	$-0.271\,382\,067\,378\,863$	21	−2	2	$0.285\,417\,173\,048\,685$
7	−8	7	$0.107\,202\,262\,490\,333 \times 10^{1}$	22	−1	0	$0.329\,924\,030\,996\,098 \times 10^{-1}$
8	−8	8	$-0.853\,821\,329\,075\,382$	23	−1	1	$0.239\,897\,419\,685\,483$
9	−6	0	$-0.215\,214\,194\,340\,526 \times 10^{-4}$	24	−1	4	$0.482\,754\,995\,951\,394 \times 10^{1}$
10	−6	1	$0.769\,656\,088\,222\,730 \times 10^{-3}$	25	−1	5	$-0.118\,035\,753\,702\,231 \times 10^{2}$
11	−6	2	$-0.431\,136\,580\,433\,864 \times 10^{-2}$	26	0	0	$0.169\,490\,044\,091\,791$
12	−6	5	$0.453\,342\,167\,309\,331$	27	1	0	$-0.179\,967\,222\,507\,787 \times 10^{-1}$
13	−6	6	$-0.507\,749\,535\,873\,652$	28	1	1	$0.371\,810\,116\,332\,674 \times 10^{-1}$
14	−6	10	$-0.100\,475\,154\,528\,389 \times 10^{3}$	29	2	2	$-0.536\,288\,335\,065\,096 \times 10^{-1}$
15	−4	3	$-0.219\,201\,924\,648\,793$	30	2	6	$0.160\,697\,101\,092\,520 \times 10^{1}$

Table 9. Selected specific volume values calculated from Eqs. (4) and (5) [a]

Equation	p / MPa	h / kJ kg^{-1}	v / m^3 kg^{-1}
$v_{3a}(p,h)$, Eq. (4)	20	1700	$1.749\,903\,962 \times 10^{-3}$
	50	2000	$1.908\,139\,035 \times 10^{-3}$
	100	2100	$1.676\,229\,776 \times 10^{-3}$
$v_{3b}(p,h)$, Eq. (5)	20	2500	$6.670\,547\,043 \times 10^{-3}$
	50	2400	$2.801\,244\,590 \times 10^{-3}$
	100	2700	$2.404\,234\,998 \times 10^{-3}$

[a] It is recommended that programmed functions be verified using 8 byte real values for all variables.

5.3 Numerical Consistency with the Basic Equation of IAPWS-IF97

The maximum temperature differences and related root-mean-square differences between the calculated temperature Eqs. (2) and (3) and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ in comparison with the permissible differences are listed in Table 10. The calculation of the root-mean-square values is described in Section 1.

Table 10 also contains the maximum relative deviations and root-mean-square relative deviations for specific volume of Eqs. (4) and (5) from IAPWS-IF97.

The critical temperature and the critical volume are met exactly by the equations $T(p,h)$ and $v(p,h)$.

Table 10. Maximum differences and root-mean-square differences of the temperature calculated from Eqs. (2) and (3) and specific volume calculated from Eqs. (4) and (5) to the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ and related permissible values

Subregion	Equation	$\lvert\Delta T\rvert_{\mathrm{tol}}$	$\lvert\Delta T\rvert_{\mathrm{max}}$	$\lvert\Delta T\rvert_{\mathrm{RMS}}$
3a	(2)	25 mK	23.6 mK	10.5 mK
3b	(3)	25 mK	19.6 mK	9.6 mK
Subregion	**Equation**	$\lvert\Delta v/v\rvert_{\mathrm{tol}}$	$\lvert\Delta v/v\rvert_{\mathrm{max}}$	$\lvert\Delta v/v\rvert_{\mathrm{RMS}}$
3a	(4)	0.01 %	0.0080 %	0.0032 %
3b	(5)	0.01 %	0.0095 %	0.0042 %

5.4 Consistency at Boundary Between Subregions

The maximum temperature difference between the two backward equations, Eq. (2) and Eq. (3), along the boundary $h_{3ab}(p)$, Eq. (1), has the following value

$$\lvert\Delta T\rvert_{\mathrm{max}} = \left| T_{3a}\left(p, h_{3ab}(p)\right) - T_{3b}\left(p, h_{3ab}(p)\right) \right|_{\mathrm{max}} = 0.37\,\mathrm{mK}\,.$$

Thus, the temperature differences between the two backward functions $T(p,h)$ of the adjacent subregions are smaller than the numerical consistencies with the IAPWS-IF97 equations.

The relative specific volume differences between the two backward equations $v(p,h)$ of the adjacent subregions 3a and 3b are also smaller than the numerical consistencies of these equations with the IAPWS-IF97 basic equation. Along the boundary $h_{3ab}(p)$, Eq. (1), the maximum difference between the corresponding equations was determined as:

$$\left|\frac{\Delta v}{v}\right|_{\max} = \left|\frac{v_{3a}\left(p, h_{3ab}(p)\right) - v_{3b}\left(p, h_{3ab}(p)\right)}{v_{3b}\left(p, h_{3ab}(p)\right)}\right|_{\max} = 0.00015\% \ .$$

6 Backward Equations $T(p,s)$ and $v(p,s)$ for Subregions 3a and 3b

6.1 The Equations $T(p,s)$

The backward equation $T_{3a}(p,s)$ for subregion 3a has the following dimensionless form:

$$\frac{T_{3a}(p,s)}{T^*} = \theta_{3a}(\pi,\sigma) = \sum_{i=1}^{33} n_i \left(\pi + 0.240\right)^{I_i} \left(\sigma - 0.703\right)^{J_i} \ , \qquad (6)$$

where $\theta = T/T^*$, $\pi = p/p^*$, and $\sigma = s/s^*$ with $T^* = 760\ \mathrm{K}$, $p^* = 100\ \mathrm{MPa}$, and $s^* = 4.4\ \mathrm{kJ\ kg^{-1}\ K^{-1}}$. The coefficients n_i and exponents I_i and J_i of Eq. (6) are listed in Table 11.

The backward equation $T_{3b}(p,s)$ for subregion 3b reads in its dimensionless form

$$\frac{T_{3b}(p,s)}{T^*} = \theta_{3b}(\pi,\sigma) = \sum_{i=1}^{28} n_i \left(\pi + 0.760\right)^{I_i} \left(\sigma - 0.818\right)^{J_i} \ , \qquad (7)$$

where $\theta = T/T^*$, $\pi = p/p^*$, and $\sigma = s/s^*$ with $T^* = 860\ \mathrm{K}$, $p^* = 100\ \mathrm{MPa}$, and $s^* = 5.3\ \mathrm{kJ\ kg^{-1}\ K^{-1}}$. The coefficients n_i and exponents I_i and J_i of Eq. (7) are listed in Table 12.

Computer-program verification

To assist the user in computer-program verification of Eqs. (6) and (7), Table 13 contains test values for calculated temperatures.

Table 11. Coefficients and exponents of the backward equation $T_{3a}(p,s)$ for subregion 3a in its dimensionless form, Eq. (6)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	28	$0.150\,042\,008\,263\,875 \times 10^{10}$	18	−4	10	$-0.368\,275\,545\,889\,071 \times 10^{3}$
2	−12	32	$-0.159\,397\,258\,480\,424 \times 10^{12}$	19	−4	36	$0.664\,768\,904\,779\,177 \times 10^{16}$
3	−10	4	$0.502\,181\,140\,217\,975 \times 10^{-3}$	20	−2	1	$0.449\,359\,251\,958\,880 \times 10^{-1}$
4	−10	10	$-0.672\,057\,767\,855\,466 \times 10^{2}$	21	−2	4	$-0.422\,897\,836\,099\,655 \times 10^{1}$
5	−10	12	$0.145\,058\,545\,404\,456 \times 10^{4}$	22	−1	1	$-0.240\,614\,376\,434\,179$
6	−10	14	$-0.823\,889\,534\,888\,890 \times 10^{4}$	23	−1	6	$-0.474\,341\,365\,254\,924 \times 10^{1}$
7	−8	5	$-0.154\,852\,214\,233\,853$	24	0	0	$0.724\,093\,999\,126\,110$
8	−8	7	$0.112\,305\,046\,746\,695 \times 10^{2}$	25	0	1	$0.923\,874\,349\,695\,897$
9	−8	8	$-0.297\,000\,213\,482\,822 \times 10^{2}$	26	0	4	$0.399\,043\,655\,281\,015 \times 10^{1}$
10	−8	28	$0.438\,565\,132\,635\,495 \times 10^{11}$	27	1	0	$0.384\,066\,651\,868\,009 \times 10^{-1}$
11	−6	2	$0.137\,837\,838\,635\,464 \times 10^{-2}$	28	2	0	$-0.359\,344\,365\,571\,848 \times 10^{-2}$
12	−6	6	$-0.297\,478\,527\,157\,462 \times 10^{1}$	29	2	3	$-0.735\,196\,448\,821\,653$
13	−6	32	$0.971\,777\,947\,349\,413 \times 10^{13}$	30	3	2	$0.188\,367\,048\,396\,131$
14	−5	0	$-0.571\,527\,767\,052\,398 \times 10^{-4}$	31	8	0	$0.141\,064\,266\,818\,704 \times 10^{-3}$
15	−5	14	$0.288\,307\,949\,778\,420 \times 10^{5}$	32	8	1	$-0.257\,418\,501\,496\,337 \times 10^{-2}$
16	−5	32	$-0.744\,428\,289\,262\,703 \times 10^{14}$	33	10	2	$0.123\,220\,024\,851\,555 \times 10^{-2}$
17	−4	6	$0.128\,017\,324\,848\,921 \times 10^{2}$				

Table 12. Coefficients and exponents of the backward equation $T_{3b}(p,s)$ for subregion 3b in its dimensionless form, Eq. (7)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	1	$0.527\,111\,701\,601\,660$	15	−5	6	$0.880\,531\,517\,490\,555 \times 10^{3}$
2	−12	3	$-0.401\,317\,830\,052\,742 \times 10^{2}$	16	−4	12	$0.265\,015\,592\,794\,626 \times 10^{7}$
3	−12	4	$0.153\,020\,073\,134\,484 \times 10^{3}$	17	−3	1	$-0.359\,287\,150\,025\,783$
4	−12	7	$-0.224\,799\,398\,218\,827 \times 10^{4}$	18	−3	6	$-0.656\,991\,567\,673\,753 \times 10^{3}$
5	−8	0	$-0.193\,993\,484\,669\,048$	19	−2	2	$0.241\,768\,149\,185\,367 \times 10^{1}$
6	−8	1	$-0.140\,467\,557\,893\,768 \times 10^{1}$	20	0	0	$0.856\,873\,461\,222\,588$
7	−8	3	$0.426\,799\,878\,114\,024 \times 10^{2}$	21	2	1	$0.655\,143\,675\,313\,458$
8	−6	0	$0.752\,810\,643\,416\,743$	22	3	1	$-0.213\,535\,213\,206\,406$
9	−6	2	$0.226\,657\,238\,616\,417 \times 10^{2}$	23	4	0	$0.562\,974\,957\,606\,348 \times 10^{-2}$
10	−6	4	$-0.622\,873\,556\,909\,932 \times 10^{3}$	24	5	24	$-0.316\,955\,725\,450\,471 \times 10^{15}$
11	−5	0	$-0.660\,823\,667\,935\,396$	25	6	0	$-0.699\,997\,000\,152\,457 \times 10^{-3}$
12	−5	1	$0.841\,267\,087\,271\,658$	26	8	3	$0.119\,845\,803\,210\,767 \times 10^{-1}$
13	−5	2	$-0.253\,717\,501\,764\,397 \times 10^{2}$	27	12	1	$0.193\,848\,122\,022\,095 \times 10^{-4}$
14	−5	4	$0.485\,708\,963\,532\,948 \times 10^{3}$	28	14	2	$-0.215\,095\,749\,182\,309 \times 10^{-4}$

Table 13. Selected temperature values calculated from Eqs. (6) and (7)[a]

Equation	p / MPa	s / kJ kg^{-1} K^{-1}	T / K
$T_{3a}(p,s)$, Eq. (6)	20	3.7	$6.208\,841\,563 \times 10^{2}$
	50	3.5	$6.181\,549\,029 \times 10^{2}$
	100	4.0	$7.056\,880\,237 \times 10^{2}$
$T_{3b}(p,s)$, Eq. (7)	20	5.0	$6.401\,176\,443 \times 10^{2}$
	50	4.5	$7.163\,687\,517 \times 10^{2}$
	100	5.0	$8.474\,332\,825 \times 10^{2}$

[a] It is recommended that programmed functions be verified using 8 byte real values for all variables.

6.2 The Equations $v(p,s)$

The backward equation $v_{3a}(p,s)$ for subregion 3a has the following dimensionless form:

$$\frac{v_{3a}(p,s)}{v^*} = \omega_{3a}(\pi, \sigma) = \sum_{i=1}^{28} n_i \left(\pi + 0.187\right)^{I_i} \left(\sigma - 0.755\right)^{J_i} \quad , \tag{8}$$

where $\omega = v/v^*$, $\pi = p/p^*$, and $\sigma = s/s^*$ with $v^* = 0.0028\ \mathrm{m^3\ kg^{-1}}$, $p^* = 100\ \mathrm{MPa}$, and $s^* = 4.4\ \mathrm{kJ\ kg^{-1}\ K^{-1}}$. The coefficients n_i and exponents I_i and J_i of Eq. (8) are listed in Table 14.

The backward equation $v_{3b}(p,s)$ for subregion 3b reads in its dimensionless form

$$\frac{v_{3b}(p,s)}{v^*} = \omega_{3b}(\pi, \sigma) = \sum_{i=1}^{31} n_i \left(\pi + 0.298\right)^{I_i} \left(\sigma - 0.816\right)^{J_i} \quad , \tag{9}$$

where $\omega = v/v^*$, $\pi = p/p^*$, and $\sigma = s/s^*$ with $v^* = 0.0088\ \mathrm{m^3\ kg^{-1}}$, $p^* = 100\ \mathrm{MPa}$, and $s^* = 5.3\ \mathrm{kJ\ kg^{-1}\ K^{-1}}$. The coefficients n_i and exponents I_i and J_i of Eq. (9) are listed in Table 15.

Computer-program verification

To assist the user in computer-program verification of Eqs. (8) and (9), Table 16 contains test values for calculated specific volumes.

Table 14. Coefficients and exponents of the backward equation $v_{3a}(p,s)$ for subregion 3a in its dimensionless form, Eq. (8).

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	10	$0.795\,544\,074\,093\,975 \times 10^{2}$	15	−3	2	$-0.118\,008\,384\,666\,987$
2	−12	12	$-0.238\,261\,242\,984\,590 \times 10^{4}$	16	−3	4	$0.253\,798\,642\,355\,900 \times 10^{1}$
3	−12	14	$0.176\,813\,100\,617\,787 \times 10^{5}$	17	−2	3	$0.965\,127\,704\,669\,424$
4	−10	4	$-0.110\,524\,727\,080\,379 \times 10^{-2}$	18	−2	8	$-0.282\,172\,420\,532\,826 \times 10^{2}$
5	−10	8	$-0.153\,213\,833\,655\,326 \times 10^{2}$	19	−1	1	$0.203\,224\,612\,353\,823$
6	−10	10	$0.297\,544\,599\,376\,982 \times 10^{3}$	20	−1	2	$0.110\,648\,186\,063\,513 \times 10^{1}$
7	−10	20	$-0.350\,315\,206\,871\,242 \times 10^{8}$	21	0	0	$0.526\,127\,948\,451\,280$
8	−8	5	$0.277\,513\,761\,062\,119$	22	0	1	$0.277\,000\,018\,736\,321$
9	−8	6	$-0.523\,964\,271\,036\,888$	23	0	3	$0.108\,153\,340\,501\,132 \times 10^{1}$
10	−8	14	$-0.148\,011\,182\,995\,403 \times 10^{6}$	24	1	0	$-0.744\,127\,885\,357\,893 \times 10^{-1}$
11	−8	16	$0.160\,014\,899\,374\,266 \times 10^{7}$	25	2	0	$0.164\,094\,443\,541\,384 \times 10^{-1}$
12	−6	28	$0.170\,802\,322\,663\,427 \times 10^{13}$	26	4	2	$-0.680\,468\,275\,301\,065 \times 10^{-1}$
13	−5	1	$0.246\,866\,996\,006\,494 \times 10^{-3}$	27	5	2	$0.257\,988\,576\,101\,640 \times 10^{-1}$
14	−4	5	$0.165\,326\,084\,797\,980 \times 10^{1}$	28	6	0	$-0.145\,749\,861\,944\,416 \times 10^{-3}$

Table 15. Coefficients and exponents of the backward equation $v_{3b}(p,s)$ for subregion 3b in its dimensionless form, Eq. (9)

i	I_i	J_i	n_i	i	I_i	J_i	n_i
1	−12	0	$0.591\,599\,780\,322\,238 \times 10^{-4}$	17	−4	2	$-0.121\,613\,320\,606\,788 \times 10^{2}$
2	−12	1	$-0.185\,465\,997\,137\,856 \times 10^{-2}$	18	−4	3	$0.167\,637\,540\,957\,944 \times 10^{1}$
3	−12	2	$0.104\,190\,510\,480\,013 \times 10^{-1}$	19	−3	1	$-0.744\,135\,838\,773\,463 \times 10^{1}$
4	−12	3	$0.598\,647\,302\,038\,590 \times 10^{-2}$	20	−2	0	$0.378\,168\,091\,437\,659 \times 10^{-1}$
5	−12	5	$-0.771\,391\,189\,901\,699$	21	−2	1	$0.401\,432\,203\,027\,688 \times 10^{1}$
6	−12	6	$0.172\,549\,765\,557\,036 \times 10^{1}$	22	−2	2	$0.160\,279\,837\,479\,185 \times 10^{2}$
7	−10	0	$-0.467\,076\,079\,846\,526 \times 10^{-3}$	23	−2	3	$0.317\,848\,779\,347\,728 \times 10^{1}$
8	−10	1	$0.134\,533\,823\,384\,439 \times 10^{-1}$	24	−2	4	$-0.358\,362\,310\,304\,853 \times 10^{1}$
9	−10	2	$-0.808\,094\,336\,805\,495 \times 10^{-1}$	25	−2	12	$-0.115\,995\,260\,446\,827 \times 10^{7}$
10	−10	4	$0.508\,139\,374\,365\,767$	26	0	0	$0.199\,256\,573\,577\,909$
11	−8	0	$0.128\,584\,643\,361\,683 \times 10^{-2}$	27	0	1	$-0.122\,270\,624\,794\,624$
12	−5	1	$-0.163\,899\,353\,915\,435 \times 10^{1}$	28	0	2	$-0.191\,449\,143\,716\,586 \times 10^{2}$
13	−5	2	$0.586\,938\,199\,318\,063 \times 10^{1}$	29	1	0	$-0.150\,448\,002\,905\,284 \times 10^{-1}$
14	−5	3	$-0.292\,466\,667\,918\,613 \times 10^{1}$	30	1	2	$0.146\,407\,900\,162\,154 \times 10^{2}$
15	−4	0	$-0.614\,076\,301\,499\,537 \times 10^{-2}$	31	2	2	$-0.327\,477\,787\,188\,230 \times 10^{1}$
16	−4	1	$0.576\,199\,014\,049\,172 \times 10^{1}$				

Table 16. Selected specific volume values calculated from Eqs. (8) and (9) [a]

Equation	p / MPa	s / kJ kg^{-1} K^{-1}	v / m^3 kg^{-1}
$v_{3a}(p,s)$, Eq. (8)	20	3.7	$1.639\,890\,984 \times 10^{-3}$
	50	3.5	$1.423\,030\,205 \times 10^{-3}$
	100	4.0	$1.555\,893\,131 \times 10^{-3}$
$v_{3b}(p,s)$, Eq. (9)	20	5.0	$6.262\,101\,987 \times 10^{-3}$
	50	4.5	$2.332\,634\,294 \times 10^{-3}$
	100	5.0	$2.449\,610\,757 \times 10^{-3}$

[a] It is recommended that programmed functions be verified using 8 byte real values for all variables.

6.3 Numerical Consistency with the Basic Equation of IAPWS-IF97

The maximum temperature differences and related root-mean-square differences between the temperatures calculated from Eqs. (6) and (7) and the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ in comparison with the permissible differences are listed in Table 17.

Table 17 also contains the maximum relative deviations and root-mean-square relative deviations for the specific volume of Eqs. (8) and (9) from IAPWS-IF97.

The critical temperature and the critical volume are met exactly by the equations $T(p,s)$ and $v(p,s)$.

Table 17. Maximum differences and root-mean-square differences of the temperature calculated from Eqs. (6) and (7), and specific volume calculated from Eqs. (8) and (9) to the IAPWS-IF97 basic equation $f_3^{97}(v,T)$ and related permissible values

Subregion	Equation	$\lvert\Delta T\rvert_{\mathrm{tol}}$	$\lvert\Delta T\rvert_{\mathrm{max}}$	$\lvert\Delta T\rvert_{\mathrm{RMS}}$
3a	(6)	25 mK	24.8 mK	11.2 mK
3b	(7)	25 mK	22.1 mK	10.1 mK
Subregion	**Equation**	$\lvert\Delta v/v\rvert_{\mathrm{tol}}$	$\lvert\Delta v/v\rvert_{\mathrm{max}}$	$\lvert\Delta v/v\rvert_{\mathrm{RMS}}$
3a	(8)	0.01 %	0.0096 %	0.0052 %
3b	(9)	0.01 %	0.0077 %	0.0037 %

6.4 Consistency at Boundary Between Subregions

The maximum temperature difference between the two backward equations, Eq. (6) and Eq. (7), along the boundary s_c, has the following value

$$\lvert\Delta T\rvert_{\max} = \lvert T_{3a}(p,s_c) - T_{3b}(p,s_c)\rvert_{\max} = 0.093\,\mathrm{mK}\,.$$

Thus, the temperature differences between the two backward functions $T(p,s)$ of the adjacent subregions are smaller than the numerical consistencies with the IAPWS-IF97 equations.

The relative specific volume differences between the two backward equations $v(p,s)$, Eqs. (8) and (9), of the adjacent subregions are also smaller than the numerical consistencies of these equations with the IAPWS-IF97 basic equation. Along the boundary s_{c}, the maximum difference between the corresponding equations was determined as

$$\left|\frac{\Delta v}{v}\right|_{\max} = \left|\frac{v_{3\mathrm{a}}(p,s_{\mathrm{c}}) - v_{3\mathrm{b}}(p,s_{\mathrm{c}})}{v_{3\mathrm{b}}(p,s_{\mathrm{c}})}\right|_{\max} = 0.00046\,\% \ .$$

7 Computing Time in Relation to IAPWS-IF97

A very important motivation for the development of the backward equations $T(p,h)$, $v(p,h)$ and $T(p,s)$, $v(p,s)$ for region 3 was reducing the computing time to obtain thermodynamic properties and differential quotients from given variables (p,h) and (p,s). In IAPWS-IF97, time-consuming iterations, *e.g.*, the 2-dimensional Newton method, are required. Using the $T_3(p,h)$, $v_3(p,h)$, $T_3(p,s)$ and $v_3(p,s)$ equations, the calculation speed is about 20 times faster than that of the 2-dimensional Newton method.

The numerical consistency of T and v obtained in this way is sufficient for most heat cycle calculations.

For users not satisfied with the numerical consistency of the backward equations, the equations are still recommended for generating starting points for the iterative process. They will significantly reduce the time required to reach the convergence criteria of the iteration.

8 References

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