FOUNDATION HEAT TRANSFER MODULE FOR *ENERGYPLUS* PROGRAM

Moncef Krarti, Pirawas Chuangchid, and Pyeongchan Ihm
University of Colorado at Boulder
CEAE Department
Boulder, 80309, USA

ABSTRACT

New calculation procedures for calculating conduction heat transfer for foundations are described. These procedures are first validated and then implemented within EnergyPlus source code (beta version 5.0). Selected results of the implementation are presented for a 3-zone low -rise building. The results indicated that the developed foundation heat transfer module accounts better than existing EnergyPlus module for the ground mass and its effects on reducing the hourly and seasonal fluctuations of slab surface temperatures.

INTRODUCTION

The energy performance of the above-grade portion of the buildings is generally well understood. Hourly prediction of heat transfer from walls exposed to ambient has helped improve the thermal efficiency of building envelopes. Unfortunately, the attention to foundation heat transfer has lagged behind other building components. Today, a quantitative understanding of foundation heat transfer is needed to accurately predict and thus improve the overall energy performance of a building. It is estimated that a basement kept uninsulated may contribute up to 60 percent of the heat loss in a tightly-sealed home that is well insulated above-grade (Labs et al., 1988).

The ground-coupling heat transfer, in almost all the existing hourly building simulation programs, is treated in a primitive way by defining a simple steady-state U-value. This crude treatment stems from the lack of a straightforward and easy approach to calculate transfer functions and/or response factors for building foundations.

Major advances in knowledge of earth-contact heat transfer have been acquired since the 1970's. Sterling and Meixel (1981), Claridge (1988), and more recently Krarti (2000) provide detailed review of the state-of-the-art ground heat transfer work. Most of the dynamic models developed for foundation heat transfer are based on numerical methods and thus require hours of computer time and are, therefore, inappropriate for use in a program like *EnergyPlus*, which is attended to perform annual simulation of the above-ground building in few minutes.

An analytical technique called the Interzone Temperature Profile Estimation (ITPE) method developed by Krarti et al. (1988) has been applied to several ground-coupling heat transfer problems. In particular, ITPE solutions have been developed to calculate heat transfer from slab-on-grade floors, basements, and earth-sheltered buildings with commonly used insulation configurations (Krarti, 1990, 1993, and 1994).

The ITPE method combines both analytical and numerical techniques to obtain two-dimensional and three-dimensional solutions of heat conduction equation. Because it is based on an analytical solution, the ITPE method handles any value of thermal insulation R-value, water table depth, and soil thermal properties. In a typical ITPE formalism, the ground (or any conductive medium) is first divided into several zones of regular shapes by “imaginary” surfaces. The geometry and the boundary conditions determine these imaginary surfaces that divide the ground medium. Then, the temperature distribution is determined in each zone by solving the heat conduction equation by an analytical technique. Along the imaginary surfaces, the temperature profiles are not known. However, these temperature profiles are determined using the heat flux continuity between the zones. For more details on the formalism and the applications of the ITPE method, refer to Krarti et al. (1990, 1993, and 1994).

Several studies compared the results from the ITPE method with measured data and with predictions based on detailed numerical solutions (Krarti et al., 1985 and 1995, and Yuill and Wray, 1987). In general, good agreement was found. While the ITPE method calculates the foundation heat flow in less than one minute of computer time, the numerical solutions require several hours of computer run time.

A study by Krarti et al. (1993) showed that the ITPE formalism can be applied to generate response factors for building foundation. These response factors can be generated in one or two minutes of computer time and are suitable for use with most hourly building simulation programs including EnergyPlus. The response factors are calculated only once by the pre-processor of the simulation program and then used to calculate ground heat fluxes at any time-step.

More recently, Chuangchid and Krarti (1998) found that three-dimensional foundation heat transfer from either buildings or refrigerated structures (with low indoor air temperatures) is independent of the foundation shape. Indeed, it was found that the foundation heat transfer depends on only one parameter characteristic of the foundation geometry. This characteristic parameter represents simply the ratio of the area over the exposed perimeter of the foundation surface. Therefore, the ITPE solutions can be applied to any arbitrary shaped foundation.

To summarize, the ITPE method is well suited to calculate hourly foundation heat transfer and provides several advantages when compared to existing methods including:

- Flexibility: The ITPE method treats a wide range of foundation types and insulation configurations.
- Accuracy: The ITPE method provides reliable foundation heat flows when compared to measured data and results from detailed numerical methods.
- Well documented: The ITPE technique formalism and applications are published in several technical journals (over 30 articles) readily available in most libraries.
- Minimal computational time: The ITPE method requires reasonable computational time to compute response factors.

In this report, the general ITPE solutions are first provided for slab and basement configurations. These solutions are the basis of the modules developed for EnergyPlus to calculate building foundation heat loss/gain. Then, the general transfer function technique is described. The transfer function is used to determine the foundation heat flux at any time step during the simulation period.

GENERAL CALCULATION PROCEDURE

Foundation heat transfer requires at least two-dimensional calculation to accurately predict the total heat loss/gain through foundations. As mention earlier, the three-dimensional foundation heat transfer can be obtained from two-dimensional heat transfer using an effective foundation half width (Chuangchid and Krarti, 1998). There are two types of foundations considered in this section: slab-on-grade floor and basement foundations.

Z-transform and transfer function

The z-transfer function technique is widely used in predicting hourly energy performance of above-grade building envelope components. The coefficients of these z-transfer functions are calculated using well established methods. This chapter outlines a new method to determine the z-transfer functions for building surfaces in contact with the ground. The developed method is based on the least-square regression technique to obtain the z-transfer function coefficients from the admittance values of the z-transfer function associated to a selected set of frequencies.

The heat flux Q(t) from a building envelope surface at a given time t, can be estimated from present and past values of indoor or outdoor surface temperatures and from the past values of heat flux:

$$Q(t) = \sum_{i=0}^{n} a_i T(t - i\Delta t) - \sum_{l=1}^{m} b_l Q(t - l\Delta t) \qquad (1)$$

with, Δt is the time step. a_i and b_l are coefficients characterizing the heat flux from the surface. Note that when the flux Q(t) is expressed only as a function of the present and past values of temperatures (i.e., b_l; l = 1, 2, …, m), the coefficients a_i are the thermal response factors as defined by Mitalas and Stephenson (1967).

Using the z-transform of equation (1), the heat flux Q(z) can be expressed as function of T(z) as follows:

$$Q(z) = \frac{\sum_{i=0}^{n} a_i z^{-i}}{1 + \sum_{l=1}^{m} b_l z^{-l}} T(z) \qquad (2)$$

or

$$\frac{Q(z)}{T(z)} = H(z) = \frac{\sum_{i=0}^{n} a_i z^{-i}}{1 + \sum_{l=1}^{m} b_l z^{-l}} \qquad (3)$$

The function $H(z)$ is known as the z-transfer function of the building envelope surface and z is the back-shift operator. In the particular case of $z = e^{j\omega\Delta t}$:

$$H(\omega) = \frac{\sum_{i=0}^{n} a_i e^{-ij\omega\Delta t}}{1 + \sum_{l=1}^{m} b_l e^{-lj\omega\Delta t}} = H_R(\omega) + jH_I(\omega) \qquad (4)$$

$H_R(\omega)$ and $H_I(\omega)$ are respectively, the real part and the imaginary part of the frequency response function $H(\omega)$.

The coefficient a_i and b_l can be determined using the error function (see details in Krarti et al., 1993). A set of linear system of equations can be expressed as follows:

$$\alpha_{i_0}^{a} - \sum_{i=0}^{n} \beta_{i_0,i}^{a} a_i + \sum_{l=1}^{m} \gamma_{i_0,l}^{a} b_l = 0 \qquad (5)$$

where

$$\alpha_{i_0}^a = \sum_{k=0}^{K} \begin{Bmatrix} H'_R(\omega_k)\cos(i_0\omega_k\Delta t) \\ -H'_I(\omega_k)\sin(i_0\omega_k\Delta t) \end{Bmatrix}$$

$$\beta_{i_0,i}^a = \sum_{k=0}^{K} \cos((i-i_0)\omega_k\Delta t)$$

$$\gamma_{i_0,l}^a = \sum_{k=0}^{K} \begin{Bmatrix} H'_R(\omega_k)\cos((l-i_0)\omega_k\Delta t) \\ -H'_I(\omega_k)\sin((l-i_0)\omega_k\Delta t) \end{Bmatrix}$$

The second equation is:

$$\alpha_{l_0}^b - \sum_{i=0}^{n} \beta_{l_0,i}^b a_i + \sum_{l=1}^{m} \gamma_{l_0,l}^b b_l = 0 \qquad (6)$$

where

$$\alpha_{l_0}^b = \sum_{k=0}^{K} \left\{ H_R'^2(\omega_k) - H_I'^2(\omega_k) \right\} \cos(l_0\omega_k\Delta t)$$

$$\beta_{l_0,i}^b = \sum_{k=0}^{K} \begin{Bmatrix} H'_R(\omega_k)\cos((l_0-i)\omega_k\Delta t) \\ -H'_I(\omega_k)\sin((l_0-i)\omega_k\Delta t) \end{Bmatrix}$$

$$\gamma_{l_0,l}^b = \sum_{k=0}^{K} \left\{ \left(H_R'^2(\omega_k) - H_I'^2(\omega_k) \right) \cos((l+l_0)\omega_k\Delta t) + 2H'_R(\omega_k)H'_I(\omega_k)\sin((l+l_0)\omega_k\Delta t) \right\}$$

Equation (30) and (31) provide a system of (n+m+1) linear equations with (n+m+1) unknowns: the coefficients a_i (n+1 unknowns) and the coefficients b_l (m unknowns). This linear system can be solved using the Gauss-Jordan elimination method. The set of frequencies ω_k is selected based on where it is most important to have accurate estimation of the transfer function. For building heat transfer for which climatic data are the driving input, the frequencies of the fundamental and harmonics of the basic daily cycle are recommended.

Based on the results of a detailed frequency analysis (Krarti et al., 1995), the total heat loss/gain through foundation can be expressed as function of the foundation surface and soil surface temperatures as follows:

$$Q(t) = \sum_{k=0}^{n} a_k T_{sf}(t-k\Delta t) - \sum_{l=1}^{m} b_l Q(t-l\Delta t) - c_m T_{sm} - c_a T_{sa}\sin(\omega t+\phi) \qquad (7)$$

where,

- a_k and b_l are the coefficients of z-transfer function H determined by regression
- c_m and c_a are the annual mean and amplitude of the foundation heat transfer U-value due to soil surface temperature variation.
- ϕ is the annual phase lag between foundation heat transfer and soil surface temperature.
- T_{sm} and T_{sa} are the annual mean and amplitude of soil surface temperature or of ambient air temperature.
- T_{sf} is the foundation indoor surface temperature.

One important finding of the frequency domain analysis (Krarti et al., 1995) is that the below grade builidng envelope reacts differently to outdoor temperature changes than do above grade surfaces. In particular, it was found that the heat flux from below grade surfaces is affected by outdoor temperature variations with cycles of more than several days. This result indicates clearly the filtering effect of the ground at high frequencies. Meanwhile, the heat flux from above grade walls is affected by outdoor temperature cycles as short as two minutes.

The analysis of Krarti et al. (1995) showed that only annual, not daily, soil surface temperature variations significantly affect ground-coupled heat flux. In contrast, both daily and annual indoor temperature variations have a large implact on foundation heat fluxes. Equation (7) translates these results in a mathematical expression that can be easily implemented in an hourly building simulation program. In most cases, only five coefficients of z-transfer functions are required to compute the hourly foundation heat flux. Typically, the knowledge of the coefficients a_0, a_1, a_2, b_1, and b_2 is sufficient for an accurate calculation of the ground-coupled heat flux. For further details on how the coefficients of Equation (7) are determined, refer to Krarti et al. (1993).

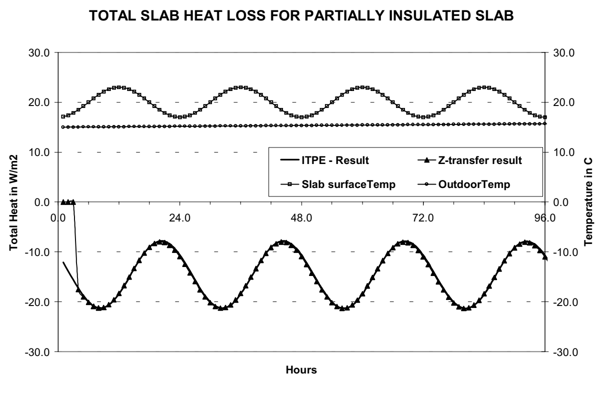

Figure 1: Heat loss from a partially insulated slab floor due to sinusoidal slab surface temperature variation

<u>Selected Validation Results</u>

Figure 1 shows the response of a slab-on-grade floor (i.e., heat flux time variation), as calculated by the z-transfer approach (Equation 1), agrees well with the results obtained directly from the ITPE solutions. Note that the heat flux calculation is unstable for the first few hours. This instability is due to the initialization requirement of the z-transfer method. In

Figure 1, the slab surface temperature is set to fluctuate as a sine function of time. Again good agreement is obtained for a different slab surface temperature fluctuation as illustrated in Figure 2. For both Figures 1 and 2, the time step of one hour is used in Equation (7) to determine the total foundation heat loss/gain.

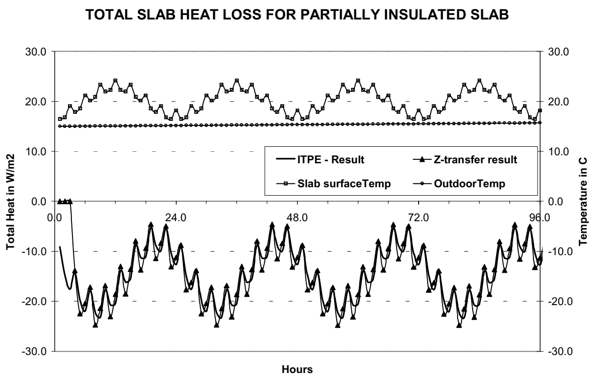

Figure 2: Heat loss from a partially insulated slab floor due to triangular slab surface temperature variation.

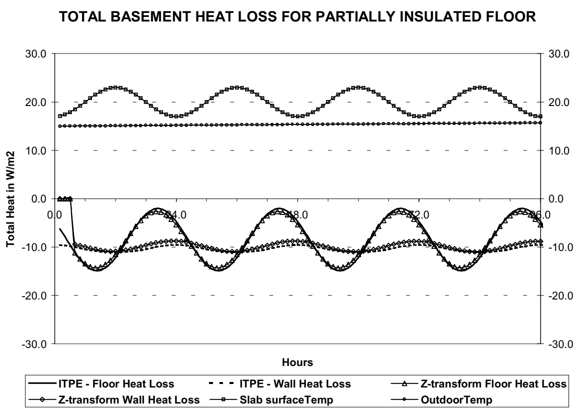

Figure 3: Heat loss from basement due to sinusoidal indoor surface temperature variation

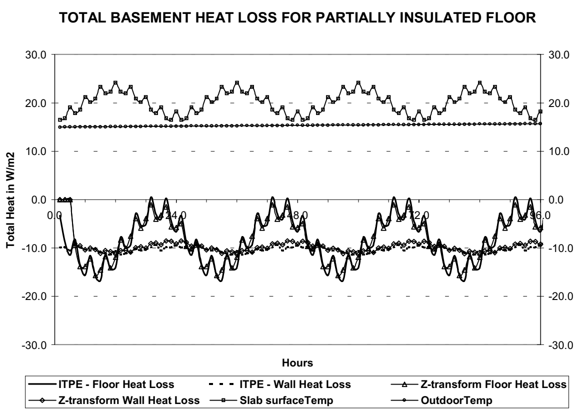

Figure 4: Heat loss from basement due triangular indoor surface temperature variation

For the case of basement foundation, Figures 3 and 4 present good agreement between the z-transfer function approach (i.e., Equation 7) and the ITPE solution for two different slab surface temperature variations. The basement configuration used in both Figures is assumed to be 10 m long and 5 m wide with a depth of 1.5 m. The wall is insulated uniformly (R-10), while the floor is partially insulated (R-10) 1 m from floor edge. The soil thermal conductivity is 1.21 W/m.K, and the soil thermal diffusivity is 4.4710^{-7} m²/s.

STRUCTURE OF THE FOUNDATION HEAT TRANSFER MODULE

This section describes the general structure of the modules that can be used to calculate foundation heat transfer in EnergyPlus. Using the approach described earlier in this report, the modules are developed based on Equation (7). The input parameters required for the module are foundation surface temperature, soil surface or ambient air (outdoor) temperature, and foundation characteristics. The output of the module is the total heat loss/gain through foundation using the z-transfer function approach described in section 2.3. More detailed description of the input/output variable as well as general structure of the procedure is presented in the next section.

Input Data

As required by EnergyPlus programming standard, the input are accomplished by means of ASCII (text) files. Specifically, there are two files: the Input Data Dictionary (IDD) and the input Data File (IDF). In the GroundHeatTransfer module, the input files are described below:

- [Type of ground contact {"Basement" or "Slab-on-grade"}] – This parameter is to notify the program about the foundation type to be simulated. The program will be then able to select the particular subroutine to calculate the total foundation heat transfer. It has to be type exactly as “Basement” or “Slab-on-grade”. Note that a crawlspace foundation can be modeled either as a basement or a slab-on-grade floor depending on the construction details.

- [Name] – less than 40 characters can be used as the name for a particular foundation.

- [Soil thermal conductivity {W/m.K}] – The soil thermal conductivity is the average value for typical soil underneath the building.

- [Soil thermal diffusivity {m^2/s}] – The soil thermal diffusivity can be determined from the heat capacity and density of soil as ($\alpha_s = \rho_s C_s / k_s$).

- [Width of Floor {m}] – For slab-on-grade floors, it is the width of slab area. For basements, the width of Floor is the width of basement floor area.

- [Length of Floor {m}] – It is the length of slab area (for slab-on-grade floors) and is the length of basement floor area (for basements).
- [Wall Height below grade {m}] – This value is the height of basement wall, and it becomes zero in the case of slab-on-grade floor foundation.
- [Length of floor perimeter insulation {m}] – For both slab-on-grade floors and basements, the insulation of the floor area can be placed along the perimeter. This value is measured from the floor perimeter area.
- [Length of wall insulation from below grade {m}] – It is the length of basement wall insulation, which is placed along the wall starting from the soil surface.
- [Overall U-value of floor {W/m².K}] – This value represents the overall conductive coefficient of floor foundation (without the indoor convective coefficient). It could be more than one material (e.g., concrete, sand gravel, etc.). This value can be obtained from the input variable specific to the building envelope, if it is available.
- [Overall U-value of floor with insulation {W/m².K}] – It is the overall conductive coefficient value that accounts for the floor insulation if the floor is insulated (without the indoor convective coefficient).
- [Overall U-value of uninsulated basement wall {W/m².K}] – This is the overall conductive coefficient for the uninsulated basement wall (without the indoor convective coefficient). This value is needed for uninsulated or partially insulated basement wall.
- [Overall U-value of insulated basement wall {W/m².K}] – It is the overall conductive coefficient of basement wall including the insulation (without the indoor convective coefficient).

Module Structure

In the main module, it contains several subroutines, which are briefly described below:

Module usage in Energy Plus (GroundHeatTrasfer Module)

GroundHeatTransfer module is the module to determine the transfer function coefficients for calculating the total foundation heat gain/loss of building. This module can be included into the main program if it is to be used.

Driver Routine (SimCTFsGround)

SimCTFsGround subroutine is called by other modules. Access to the module and its data elements are only allowed through this subroutine. It is the only "PUBLIC" routine in the module since it is accessed from outside of the module. All other routines in this module are accessed from the main driver routines.

Get Routine (GetCTFsGroundInput)

The input file is first read by this subroutine called GetCTFsGroundInput.

Initialization and Renaming Routine (InitCTFsGround)

This subroutine is used to initialize and rename all variables for the program. The module has specific names that are different from those used in other parts of EnergyPlus program.

Calculate Routine (CalculateCTFsGround)

This subroutine is considered as the main subroutine of the simulation module. It contains all necessary subroutines to determine transfer coefficients for the total foundation heat gain/loss. Specifically, this main subroutine provides the transfer function coefficients of total foundation heat loss/gain using indoor and outdoor temperatures. It is based on z-transfer function, which requires separate calculations for each type of foundation. These calculations are performed using semi-analytical solutions as described section 2.

Update Routine (UpdateCTFsGround)

UpdateCTFsGround subroutine transfers output data from GroundHeatTransfer module to other modules within EnergyPlus.

Module for Matrix Solver

LAPACK solver (SIAM, 1994) is selected to solve linear system in this module. It should be noted that LAPACK solver was written in FORTRAN 77. Therefore, some parts of LAPACK solver have been rewritten to be compatible with the free format as FORTRAN 90.

Interface Subroutine to Energy Plus

To use the GroundHeatTransfer module in EnergyPlus, "GroundHeatTransfer" has to be included in USE statement of that subroutine. Then "SimCTFsGround" is called to calculate and update the transfer coefficient in DataGroundHeatTransfer. Then, the total foundation heat transfer is determined by using the z-transform equations presented in section 2.

IMPLEMENTATION INTO ENERGYPLUS

The Foundation Heat Transfer Module developed above has been implemented into EnergyPlus source code (Beta version 5.0).

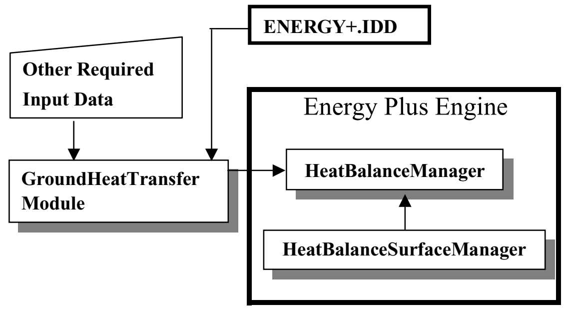

Figure 5: Flow Chart of Implementation of ITPE Ground Coupled Heat Transfer Module

Figure 5 shows the flow chart of the implementation procedure. Most of the required input data are part of the Energy+.idd file. However, some other input data (such as partial insulation description as well as annual mean and amplitude of ambient temperature) are defined within the GroundHeatTransfer Module. To fully implement the GroundHeatTransfer Module, changes of various modules within the EnergyPlus Engine had to be performed which made the implementation task difficult to execute since the entire code has to be evaluated. In particular, two modules within EnergyPlus Engine have to be modified namely: HeatBalanceManager and HeatBalanceSurfaceManager as indicated in Figure 5. These changes are detailed in section 4.2 below. In particular, Equation (27) has been implemented in HeatBalanceManger module.

It should be noted that refinements of the GroundHeatTransfer Module implementation may be needed to make all the input data required for foundation heat transfer calculation available by the EnergyPlus input file.

SELECTED ENERGYPLUS RESULTS

The foundation heat transfer module implemented in EnergyPlus has been tested for various configurations using first one-zone building. After this preliminary test, a building with several zone has been considered. Figure 6 illustrates 3-zone building (for which input file is provided with EnergyPlus source code as one of three samples) used for testing the implementation of the foundation heat transfer module.

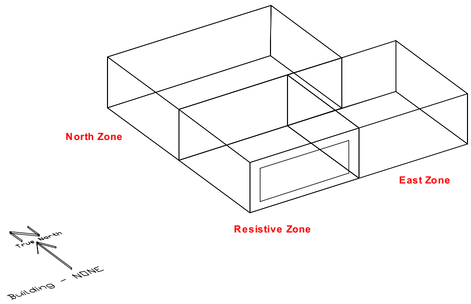

Figure 6: 3-zone building with slab-on-grade foundation.

Various output parameters are checked to assess the validity of the results. In particular, slab surface, indoor ambient temperature, as well as total building heating and cooling loads are evaluated based on hourly basis then on daily and monthly basis. Figures 7 and 8 illustrate some monthly results obtained with EnergyPlus with and without the developed foundation heat transfer module. Typically, the results show that the developed foundation heat transfer module accounts for higher thermal mass of the ground since slab surface temperature has a slight less variation over the year. However, the contribution of the ground heat transfer in the total heating and cooling loads is generally small. During the swing months, the effects of the foundation heat transfer are more noticeable for the heating loads in the Resistive Zone due most likely to higher slab surface temperatures.

Based on preliminary implementation results, the effects of the developed foundation module is more significant when the indoor temperature is allowed to float (for instance in crawlspace and unheated basements).

In terms of runtime, the foundation module doubles the CPU time of EnergyPlus for the one-zone and three-zone buildings considered in the implementation tests. Some efforts have been tried to reduce this runtime with much success. A new approach is being tested to develop an equivalent one-dimensional floor construction. This approach is somewhat similar to the existing method to model foundation heat transfer. However, the GroundHeatTransfer Module would determine the characteristics (thermal properties and thickness) of the equivalent floor construction so that the floor has the same Conduction Transfer Function (CTF) than the actual building foundation. The determination of the equivalent construction requires some robust optimization techniques.

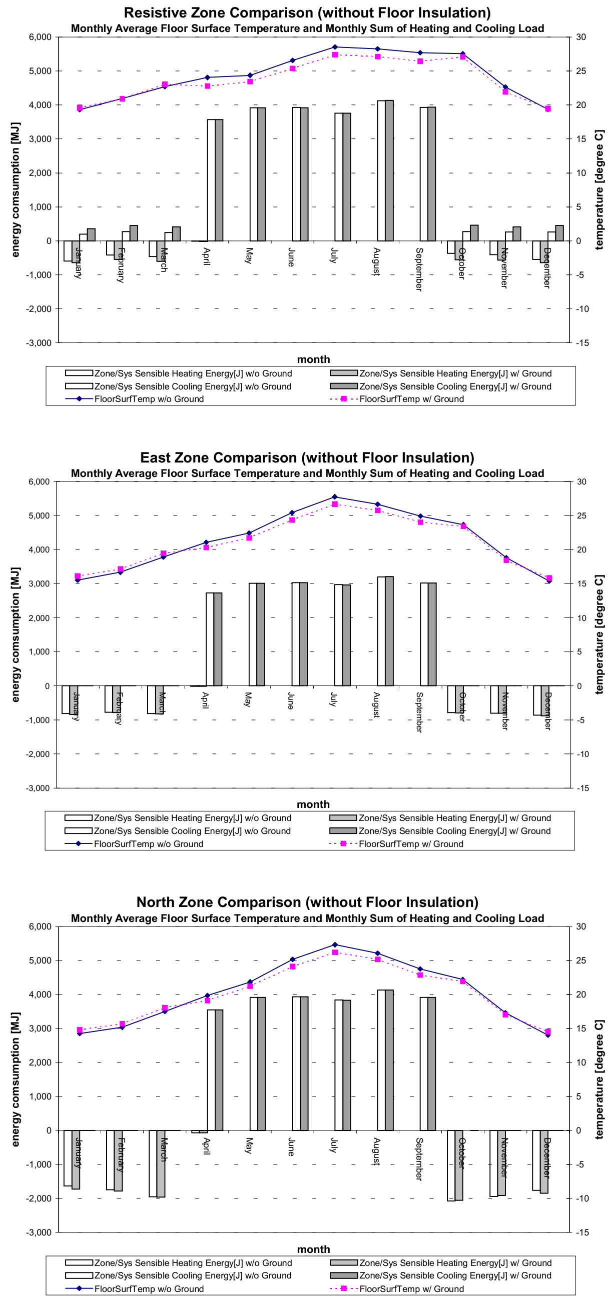

Figure 7: Comparison of selected EnergyPlus results obtained with and without ITPE foundation heat transfer Module for 3-zone building with uninsulated slab-on-grade foundation.

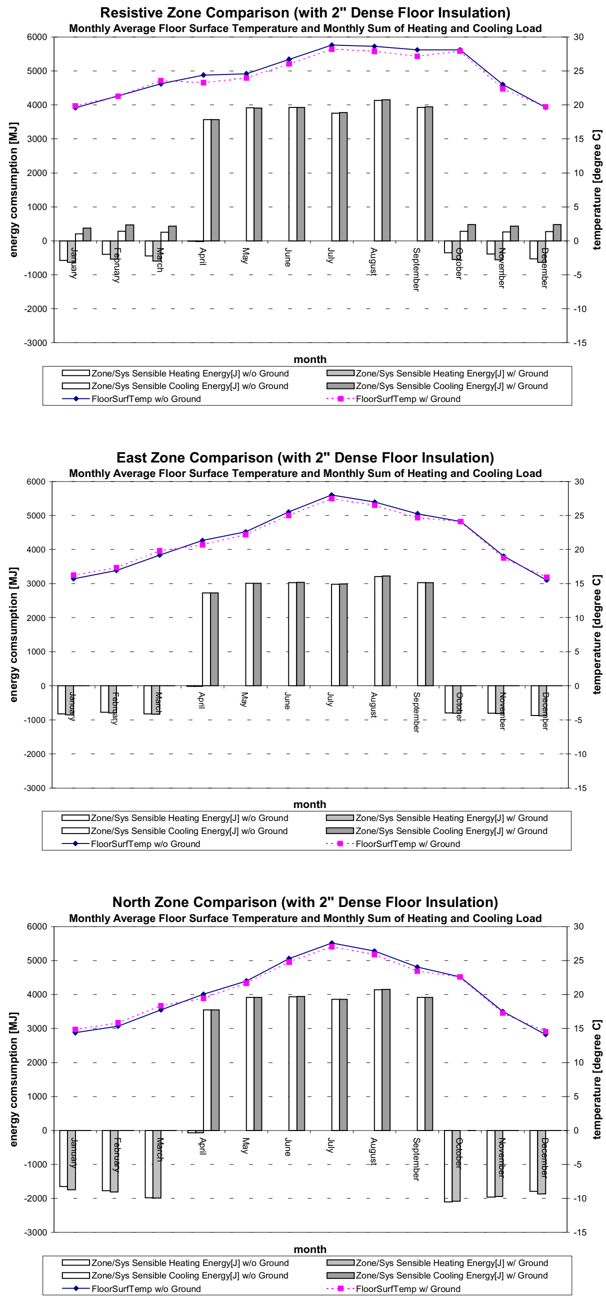

Figure 8: Comparison of selected EnergyPlus results obtained with and without ITPE foundation heat transfer Module for 3-zone building with uninsulated slab-on-grade foundation.

CONCLUSIONS

A module for the calculation of the transfer function coefficients of foundation heat transfer is developed and implemented in EnergyPlus. The construction materials and the geometric characteristics of the foundation are required as inputs for the module. Two foundation types are considered in the module: slab-on-grade floor and rectangular basements. However, crawlspaces can be modeled as an unconditioned space using one of the two foundation types.

The major issue to be addressed in refining the developed foundation heat transfer module is a reduction in runtime. Even though, the foundation heat transfer module is called only once for a given zone to compute heating and cooling loads, it almost doubles the runtime of EnergyPlus. A new approach to determine an equivalent one-dimensional floor construction is being tested and will be further assessed to evaluate its potential in reducing CPU time for running EnergyPlus.

REFERENCES

Chuangchid, P., and Krarti, M. [1998]. Comparative Analysis of Three-dimensional Foundation Heat Transfer from Slab-on-Grade Floor Slabs. Proceedings of ASME Solar Energy Engineering. pp. 37-44.

Claridge, D., 1988, *Design Methods for Earth-Contact Heat Transfer. Progress in Solar Energy* (Edited by K. Boer), American Solar Energy Society, Boulder, CO.

Labs, K., Carmody, J., Sterling, R., Shen, L., Huang, Y. and Parker, D., 1988, *Building Foundation Design Handbook*, ORNL Report Sub/86-72143/1. Oak Ridge, TN: Oak Ridge National Laboratory.

Krarti, M., Claridge, D.E., and Kreider, J.F., 1985, "The Interzone Temperature Profile Estimation, Slab-on-grade Heat Transfer Results," *Proceedings of the 23rd National Heat Transfer Conference*, Vol. 31, pp. 11-20.

Krarti, M., Claridge, D.E., and Kreider, J.F., 1988, "ITPE technique applications to time varying two-dimensional ground-coupling problems," *Int. J. Heat Mass Transfer*, Vol.31(9), pp. 1899-1911.

Krarti, M., 1989, "Steady-state heat transfer beneath partially insulated slab-on-grade floor," *Int. J. Heat Mass Transfer*, Vol. 32 (5), pp. 961-969.

Krarti, M., and Claridge, D.E., 1990, "Two-dimensional heat transfer from earth-sheltered buildings," *J. of Solar Energy Engineering*, Vol. 112, pp. 43-50.

Krarti, M., Claridge, D.E., and Kreider, J.F., 1990, "The ITPE Method Applied to Time-Varying Three-dimensional Ground-coupling Problems," *ASME J. of Heat Transfer*, Vol. 112, No. 4, pp. 849-856.

Krarti, M., Claridge, D.E., and Kreider, J.F., 1993, *Energy Calculations for Basements, Slabs, and Crawl Space,* ASHRAE TC 4.7 Project 666-RP, pp. 69-97.

Krarti, M., 1994, "Steady-state heat transfer from horizontally insulated slabs," *Int. J. Heat Mass Transfer*, Vol. 36 (8), pp. 2135-2145.

Krarti, M., Claridge, D.E., and Kreider, J.F., 1995, "Frequency Response Analysis of Ground-coupled Building Envelope Surfaces," *ASHRAE Transactions*, Vol. 101, pp. 355.

Krarti, M., and Choi, S., 1996, "Simplified Method for Foundation Heat Loss Calculation," *ASHRAE Transactions*, Vol. 102, pp. 140-152.

Krarti, M., 2000, "Foundation Heat Transfer," Chapter in *Advance in Solar Energy*, Vol. 13, editor Y. Goswami and K. Boer, ASES, Boulder, CO.

Mitalas, G.P., and Stephenson, D.G., 1967, "Room Thermal Response Factors," *ASHRAE Transactions*, Vol. 73, Part 1, pp. II.2.1-II.27.

SIAM, 1994, Lapack User's Guide, V. 2.0, Philadelphia, P.A.

Sterling, R. and Meixel, G., 1981, *Review of Underground Heat Transfer Research*, Earth Sheltered Performance and Evaluation Proceedings, Second Technical Conference (Edited by L. Boyer), pp. 67-74, Oklahoma State University.

Yuill, G.K., and Wray, C.P., 1987, "Verification of a Microcomputer Program Implementing the Mitalas Below-Grade-Heat Loss Model", *ASHRAE Transactions*, Vol. 93, part 1, pp. 434-446.