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TRNSYS Type

Vertical Borehole Heat Exchanger
EWS Model

Version 2.4

Model description and implementation in TRNSYS

Developed in the project

Low Temperature Low Cost Heat Pump Heating System

carried out by the Information Center for Electricity Applications

under contract of the Swiss Federal Office of Energy

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Type250: Vertical Borehole Heat Exchanger, EWS Model

General description

With this TRNSYS type, vertical borehole heat exchangers with double-U-pipes can be simulated. They are normally used as heat sources for ground coupled heat pumps. But it is also possible to use them directly (without a heat pump) in air-conditioning systems for cooling purposes.

To simulate heating systems with heat pumps, it is very important, that short time steps can be simulated and the transient behavior is calculated properly, since most of these heat pumps are controlled by turning the pump on and off. Measurements have shown that the start up losses (cycling losses) of heat pumps normally cannot be neglected. Therefore it is also important that the model of the heat source is able to predict the transient behavior correctly. Furthermore, a PC should take not more than a few minutes of computational time for the simulation of a whole year.

The problem can be solved by a simulation of the transient heat flux in the earth within a radius of about 2 m around the borehole with the Crank-Nicholson algorithm. In the vertical direction, the earth is divided into several, horizontal layers. Each layer can have thermal properties of its own. The brine is simulated dynamically to take into account the transient behavior when starting up. For the outer boundary condition, the analytical formula of Werner [5] for constant heat extraction could be adapted for the present problem. This formula belongs to a group of analytical solutions first described by Kelvin in his line source theory. By superposing constant heat extractions, starting at different time steps, it is possible to calculate the temperature profile at the outer boundaries of the simulation area and even to predict properly the refilling of the temperature sink in the summer.

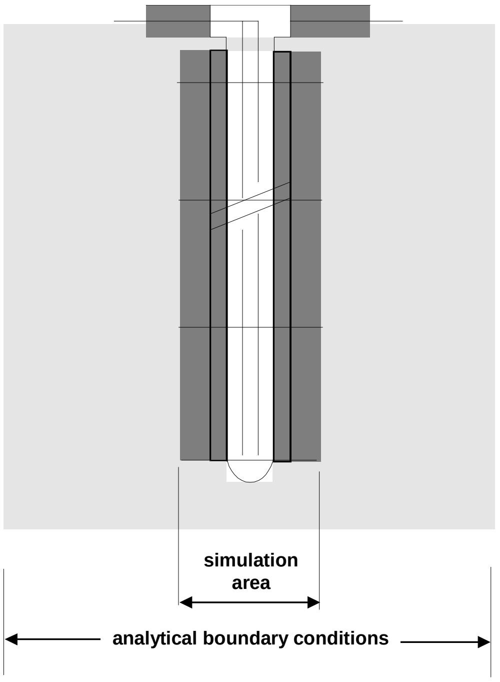

Fig. 1: Simulation of the earth next to the borehole with Crank-Nicholson schema and analytical outer boundary conditions with an adaptation of the formula of Werner [5]

A more comprehensive description of the used models and a comparison of calculations with measurements can be found in [2]. In these comparisons the transient behavior was investigated as well as the long term behavior over a period of 4½ years. They showed very good correspondence between calculation and measurement. To get such a good result, though, it is crucial to know the thermal properties of the ground and the temperature profile at the beginning of the simulation period. The best model cannot help, if they are not known. Usually a good guess for the thermal properties can be found in [4].

Symbols

Variables

α_o	Heat transfer coefficient from the brine to the pipes at non operating mode
α_1	Heat transfer coefficient from the brine to the pipes, when the pump is running
Δ	Difference
λ	Thermal conductivity
ν	Kinematic viscosity
ρ	Density
ξ	Friction factor
C	Heat capacity
c	specific heat capacity
D_b	Borehole diameter
D_i	Inner diameter of the pipes
dl	Length of a borehole element
dt	Internal time step to calculate the earth
dt2	Internal time step to calculate the brine
f	grid factor in radial direction
L	Thermal conductance
L_0	Thermal conductance of the flowing brine in vertical direction
L_1	Thermal conductance between the brine and the filling material
m	mass of the brine in the element dl in 2 pipes
$\dot{q}$	Specific heat extraction
r	Radial distance from the borehole axis
r_o	Inner radius of the pipes
r_1	Radius of the borehole
r_m	Radius of the outer boundary of the simulation area
rz	Radial center of gravity
R	Thermal resistance
R_a	Internal thermal resistance
R_b	Borehole thermal resistance
t	time
T	Temperature
T_b	Borehole temperature
TEarth	Temperature of the earth
TDown	Temperature of the downward flowing brine
TUp	Temperature of the upward flowing brine
TSource	Source temperature (brine coming out of the vertical borehole heat exchanger)
TSink	Brine temperature at the inlet of the vertical borehole heat exchanger
v	Brine velocity in the pipes

Indices

DimAxi	Number of grid points in axial direction
DimRad	Number of grid points in radial direction
i	Axial coordinate
j	Radial coordinate
k	time coordinate
lam	Laminar
p	constant pressure
turb	Turbulent
Erde	Earth
Fill	Filling material
Sole	Brine
t	Time
Woche	week of focus for the outer boundary conditions

Dimensionless Numbers

Nu	Nusselt Number
Pr	Prandtl Number
Re	Reynolds Number

Mathematical description

The Crank - Nicholson schema

In radial direction the one-dimensional heat equation or Fourier equation has to be solved:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_p} \frac{\partial^2 T}{\partial x \partial x} \quad \text{with } T = T(t,x)$$

Eq. 1

As an implicit equation of differences it is written as:

$$T_{k+1,j} - \frac{dt}{2}\frac{L_j}{C_j}\left(T_{k+1,j-1} - T_{k+1,j}\right) - \frac{dt}{2}\frac{L_{j+1}}{C_j}\left(T_{k+1,j+1} - T_{k+1,j}\right) =$$

$$T_{k,j} + \frac{dt}{2}\frac{L_j}{C_j}\left(T_{k,j-1} - T_{k,j}\right) + \frac{dt}{2}\frac{L_{j+1}}{C_j}\left(T_{k,j+1} - T_{k,j}\right)$$

Eq. 2

Index k belongs to the time coordinate and index j to the radial coordinate. C is the capacity which is described below. L is the conductance, the reciprocal of a resistance:

$$L = \frac{1}{R} = \frac{\dot{Q}}{\Delta T}$$

Eq. 3

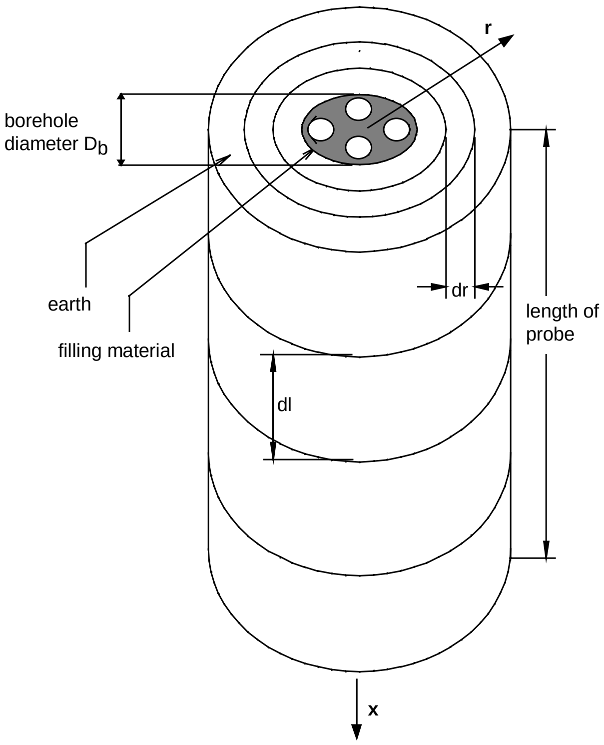

Fig. 2: Cylindrical coordinate system to solve the one-dimensional heat equation for each axial layer, with thermal properties of its own in each layer.

Arithmetical grid

In axial (vertical) direction, the borehole heat exchanger and the adjacent earth are divided into equidistant layers of length

$$dl = \frac{\text{borehole length}}{\text{DimAxi}}$$

Eq. 4

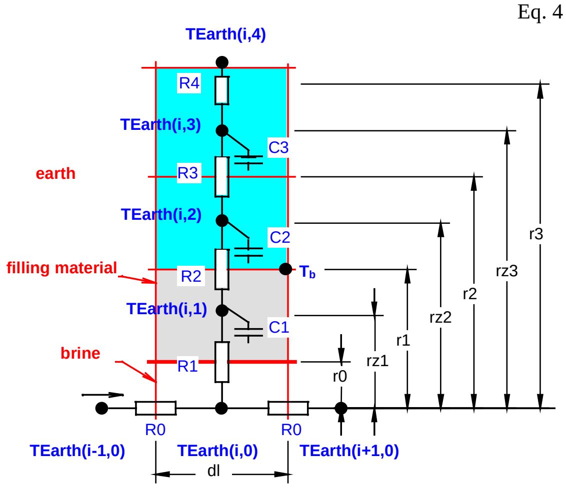

Fig. 3: Overview of the naming in a vertical layer

The grid in radial direction is variable. It is defined by the grid factor

$$\text{grid factor } f = \frac{r_{j+1} - r_j}{r_j - r_{j-1}}$$

Eq. 5

A grid factor 2 doubles the difference of the radiuses of two neighboring calculation volumes.

The simulation area is defined by pre-setting a maximum radius. The grid is given then by the following expression:

$r_0 = D_i/2$ = inner radius of the pipes
$r_1 = D_b/2$ = radius of the borehole
r_m = maximum radius of the simulation area

$$j \geq 2: \quad r_j = r_{j-1} + (r_m - r_1)\frac{1-f}{1-f^{m-1}} f^{j-2}$$

Eq. 6

Definition of capacities and resistances

Heat capacities

Heat capacities are defined for the filling material and for all layers of the surrounding ground. The heat capacity of the pipe wall is ignored:

$$C_1 = c_{\mathrm{p,\,Fill}}\,\rho_{\mathrm{Fill}}\,\pi(r_1^2 - 4\ \ r_0^2)dl$$
$$C_2 = c_{\mathrm{p,\,Erde}}\,\rho_{\mathrm{Erde}}\,\pi(r_2^2 - r_1^2)dl$$
$$C_3 = c_{\mathrm{p,Erde}}\,\rho_{\mathrm{Erde}}\,\pi(r_3^2 - r_2^2)dl$$

Eq. 7

Thermal resistances

The heat resistances of the filling and the ground are:

$$R_1 = \frac{1}{4}\left(\frac{1}{2\,\pi\,\alpha\,r_0\ dl} + \frac{1}{2\,\pi\,\lambda_{\mathrm{Fill}}\ dl}\ln\frac{r_1 - rz_1}{r_0}\right)$$

Eq. 8

$$R_2 = \frac{1}{2\,\pi\ dl}\left(\frac{1}{\lambda_{\mathrm{Fill}}}\ln\frac{r_1}{rz_1} + \frac{1}{\lambda_{\mathrm{Erde}}}\ln\frac{rz_2}{r_1}\right)$$

Eq. 9

$$R_3 = \frac{1}{2\,\pi\ dl}\frac{1}{\lambda_{\mathrm{Erde}}}\ln\frac{rz_3}{rz_2}$$

Eq. 10

$$R_4 = \frac{1}{2\ \ \pi\,\lambda_{Erde}\ dl}\ln\frac{r_3}{rz_3}$$

Eq. 11

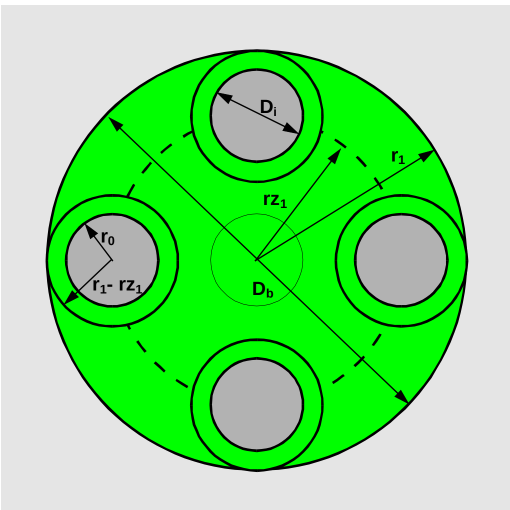

Fig. 4: Double-U-pipe borehole system

R_3 and R_4 can be obtained analytically. With R_1 and R_2 this is not possible, since we do not know the precise location of the pipes in the borehole. So far, we assumed that they are rather peripherally located. But the user of the present TRNSYS-Type is free to use any other value for R1, since R1 can optionally be set as an input parameter. As a third possibility, we can use the internal thermal resistance

$$R_a = \ \ 4\ \ dl\ \ R_1$$

Eq. 12

and the borehole thermal resistance

$$R_b = \frac{dl\ \ (T_{Sole}\ \ -\ \ T_b)}{\dot{Q}}$$

Eq. 13

as they were defined by Hellström [1].

If only R_b is given instead of R_1, then R_1 can be calculated with the following equation:

$$R_1 = \frac{R_b}{dl}\ \ -\ \ \frac{1}{2\ \ \pi\ \ \lambda_{Fill}\ \ dl}\ \ln\frac{r_1}{rz_1}$$

Eq. 14

If R_a and R_b are given as parameters, then R_1 can be calculated with

$$R_1 = \frac{R_a}{4\ \ dl}$$

Eq. 15

and R_2 with

$$R_2 = \frac{(R_b - \frac{R_a}{4})}{dl} + \frac{1}{2\,\pi\,dl}\frac{1}{\lambda_{\mathrm{Erde}}}\ln\frac{rz_2}{r_1}$$

Eq. 16

With the input parameter calcBTR the preferred option can be chosen by the user :

calcBTR	R_1	R_2
1	Eq. 8	Eq. 9
2	R_1 given as input	Eq. 9
3	Eq. 14	Eq. 9
4	Eq. 15	Eq. 16

Solving the equations

Eq. 2 can be rewritten as a matrix equation:

$$[A]\cdot\{T\}_i^{k+1}=[F]\cdot\{T\}_i^{k}$$

Eq. 17

To find the new temperature field, the Matrix A has to be inverted

$$\{T\}_i^{k+1}=[B]\cdot\{T\}_i^{k}$$

Eq. 18

where B is defined by:

$$[B]=[A]^{-1}\cdot[F]$$

Eq. 19

Non steady-state calculation of the brine

The brine temperature is used as the inner boundary condition for the simulation of the earth with the Crank-Nicholson schema. If we set the flag *Stationaer = 0* then the brine temperature is calculated with an explicit, non steady-state time-step method.

The velocity of the brine in the pipes can be calculated with the mass flow rate:

$$v = \frac{\dot{m}}{2\,\pi\ r_0^2\ \rho_{Sole}}$$

Eq. 20

As in the radial direction, we can define a thermal conductance in axial direction:

$$L_0 = c_{p,Sole}\,\dot{m} = 2\pi\ r_0^2\,v\,\rho_{Sole}\,c_{p,Sole}$$

Eq. 21

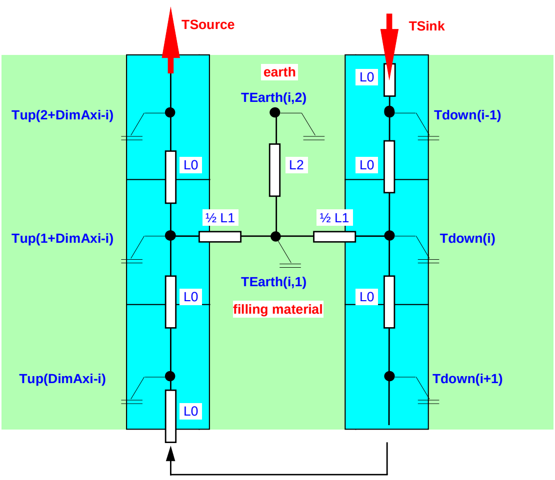

Fig. 5: Non steady-state simulation of the brine with an explicit time-step method

Now we calculate the energy balance for the upward and the downward flowing brine in a vertical layer. To simplify the calculations, we combine the two pipes in which the brine flows in the same direction and treat them as a single element for computational purposes. The mass of this element is then

$$m = 2\,\pi\,r_0^2\ dl\ \rho_{Sole}$$

Eq. 22

The energy balance for such an element gives:

$$Tdown_{k+1,i} = Tdown_{k,i} + \left(Tdown_{k+1,i-1} - Tdown_{k,i}\right)\frac{L_0\ dt2}{m\ cp} + \left(TEarth_{k,i,1} - Tdown_{k,i}\right)\frac{L_1\ dt2}{2\ m\ cp}$$

Eq. 23

and in the upward direction

$$Tup_{k+1,i} = Tup_{k,i} + \left(Tup_{k+1,i-1} - Tup_{k,i}\right)\frac{L_0\ dt2}{m\ cp} + \left(TEarth_{k,1+DimAxi-i,1} - Tup_{k,i}\right)\frac{L_1\ dt2}{2\ m\ cp}$$

Eq. 24

with the boundary condition

$$Tdown_{k+1,0} = TSink$$

Eq. 25

$$Tup_{k+1,0} = Tdown_{k+1,DimAxi}$$

Eq. 26

$$TSource = Tup_{k+1,DimAxi}$$

These equations have to be solved in direction of the flowing brine.

Steady-state calculation of the brine

As an option, a steady-state calculation can be carried out for the brine. To do so, we set the input parameter *Stationaer = 1*. Then the energy balance gives us

$$Tdown_i = \frac{\left(L_0\ Tdown_{i-1} + \frac{L_1}{2}\ TEarth_{i,1}\right)}{\left(L_0 + \frac{L_1}{2}\right)}$$

Eq. 27

and

$$Tup_i = \frac{\left(L_0\ Tup_{i-1} + \frac{L_1}{2}\ TEarth_{1+DimAxi-i,1}\right)}{\left(L_0 + \frac{L_1}{2}\right)}$$

Eq. 28

Outer boundary condition

For the outer boundary condition, the analytical formula of Werner [5] for constant heat extraction can be adapted for the present problem. This formula belongs to a group of analytical solutions first described by Kelvin in his line source theory. By superposing constant heat extractions, starting at different time steps, it is possible to calculate the temperature profile at the outer boundaries of the simulation area and even to predict the refilling of the temperature sink in the summer properly.

The temperature drop in the earth in function of the distance from the borehole and time can be written as:

$$\Delta T(r,t) = \frac{\dot{q}}{4\,\pi\,\lambda} W(u)$$

Eq. 29

with

$$W(u) = \left[-0.5772 - \ln(u) + u - \frac{u^2}{2\,2!} + \frac{u^3}{3\,3!} - \frac{u^4}{4\,4!} + \ldots \right]$$

Eq. 30

and

$$u(r,t) = \frac{c_{p,Erde}\ \rho_{Erde}}{4\,t\ \lambda_{Erde}} r^2$$

Eq. 31

In these formulas, the specific heat extraction

$$\dot{q} = \frac{\dot{Q}}{\text{borehole length}}$$

Eq. 32

has to be constant. Since in real boreholes the heat extraction is not constant, we must superpose different constant heat extractions q, starting at different time steps to get a variable heat extraction:

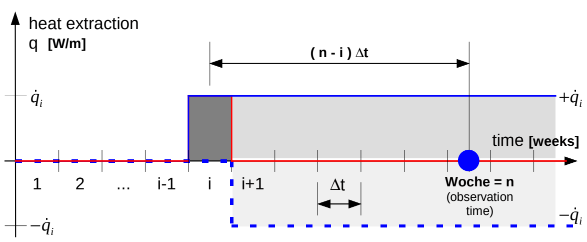

Fig. 6: To get a discrete heat extraction q_i we superpose q and -q, starting at different time steps.

To get the temperature drop at the time (n·Δt) we have to add all the effects of this constant heat extractions in the following way:

$$\Delta T(r, t = n\,\Delta t) \;=\; \sum_{i=1}^{n} \frac{W(u(r, t = i\,\Delta t))}{4\,\pi\,\lambda} \left[\dot{q}_{n-i+1} - \dot{q}_{n-i} \right]$$

Eq. 33

with

$$\dot{q}_0 = 0$$

Eq. 34

Thus the temperature at the outer boundary of the simulation area can be written as:

$$TEarth(DimRad + 1) = T_0(i) - \Delta T(r = r_{DimRad})$$

Eq. 35

Of course this has to be calculated for each vertical layer with its own specific heat extraction rate.

These outer boundary conditions are calculated weekly and then held constant during the whole week.

Heat transfer coefficient

The heat transfer coefficient from the brine to the pipes can be calculated from the Nusselt Number:

$$\alpha_1 = \frac{Nu(\mathrm{Re}, \mathrm{Pr})\ \lambda_{Sole}}{D_i}$$

Eq. 36

When we have laminar flow (Re < 2'300), the Nusselt Number is taken constant:

$$Nu_{lam} = 4.36$$

Eq. 37

With turbulent flow (Re > 10'000), the Petukhov Formula [3] is used:

$$Nu_{turb} = \frac{\frac{\xi}{8}}{K_1 \;+\; K_2 \sqrt{\frac{\xi}{8}} \left(\mathrm{Pr}^{2/3} \;-\; 1 \right)} \ \mathrm{Re}\ \ \mathrm{Pr}$$

Eq. 38

with

$$K_1 = 1 \;+\; 27.2 \left(\frac{\xi}{8} \right)$$

Eq. 39

$$K_2 = 11.7 \;+\; 1.8\ \ \mathrm{Pr}^{-1/3}$$

Eq. 40

According to the recommendations of Merker [3], the friction factor ξ for turbulent flow can be calculated with

$$\xi = \frac{1}{\left(1.82\log(\mathrm{Re}) - 1.64\right)^2}$$

Eq. 41

In the transition laminar - turbulent (2'300 < Re < 10'000) we use

$$Nu = Nu_{lam}\exp\left[\ln\left(\frac{Nu_0}{Nu_{lam}}\right)\frac{\ln\left(\frac{\mathrm{Re}}{2'300}\right)}{\ln\left(\frac{10'000}{2'300}\right)}\right]$$

Eq. 42

with

$$Nu_0\ (Re = 10'000) = \frac{\frac{\xi_0}{8}}{1.107 + K_2\sqrt{\frac{\xi_0}{8}}\left(\mathrm{Pr}^{2/3} - 1\right)}\ \mathrm{Re}\ \mathrm{Pr}$$

Eq. 43

$$\xi_0 = 0.031437$$

Eq. 44

When the pump is not running, we use the following heat transfer coefficient:

$$\alpha_0 = \frac{\lambda_{Sole}}{\frac{D_i}{2}\left(1 - \sqrt{0.5}\right)}$$

Eq. 45

The heat transfer coefficient is only used, when R_1 is calculated internally. Otherwise it is already included in the first thermal resistance.

Flow chart

The present model is calculating internally with smaller timesteps. These are optimized in the code, so the user does not have to be concerned with them.

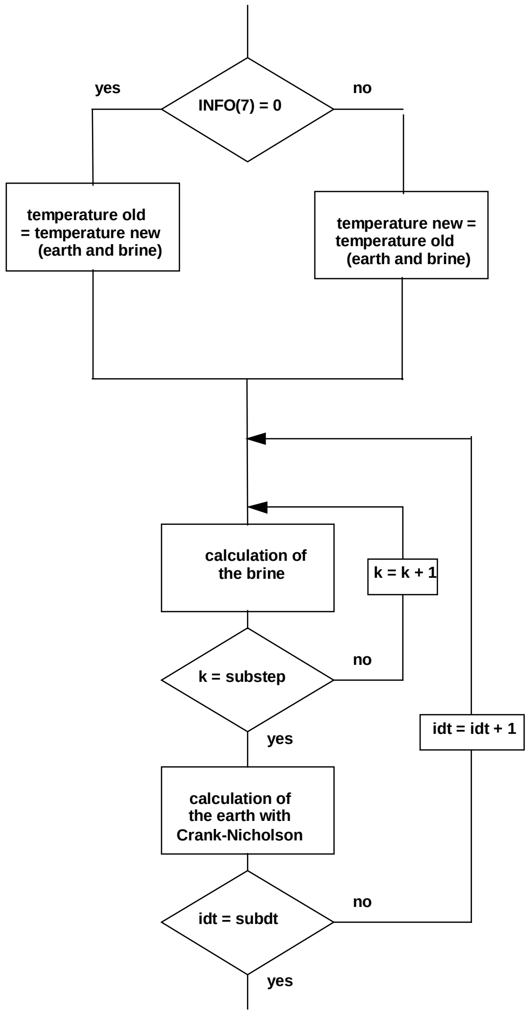

Fig. 7: Flow chart of the code.

Component configuration

Parameter	Fortran variable	Description
1	Stationaer	Flag to set the calculation mode of the brine (1: steady-state, 0: non steady-state)
2	calcBTR	Flag to specify the means of calculating the thermal resistances of the borehole: 1: calculate R_1 and R_2 internally 2: R_1 is an input parameter R_2 is calculated internally 3: R_1=f(R_b), R_2 is calculated internally 4: R_1=f(R_a), R_2=f(R_a,R_b)
3	Auslegungsmassenstrom	Mass flow rate of the brine, used to calculate the heat transfer coefficient α_1
4	Rechenradius	Radius of the outer boundary r_m of the simulation area
5	Gitterfaktor	Grid factor f
6	Sondenlaenge	Length of the borehole
7	Sondendurchmesser	Inner diameter of the pipes D_i
8	Bohrdurchmesser	Borehole Diameter D_b
9	TGrad	Axial temperature gradient in the earth at the start of the simulation
10	Jahresmitteltemp	Average annual air temperature
11	Bodenerwaermung	Average yearly temperature difference between the soil surface and the air
12	cpFill	Specific heat capacity of the filling material
13	rhoFill	Density of the filling material
15	lambdaFill	Heat conductivity of the filling material
15	cpSole	Specific heat capacity of the brine
16	rhoSole	Density of the brine
17	lambdaSole	Heat conductivity of the brine
18	nueSole	Kinematic viscosity of the brine
19	MonitorAxi1	Axial coordinate of the 1st monitor temperature
20	MonitorRad1	Radial coordinate of the 1st monitor temperature
21	MonitorAxi2	Axial coordinate of the 2nd monitor temp.
22	MonitorRad2	Radial coordinate of the 2nd monitor temp.
23	MonitorAxi3	Axial coordinate of the 3rd monitor temperature
24	MonitorRad3	Radial coordinate of the 3rd monitor temperature
25	MonitorAxi4	Axial coordinate of the 4th monitor temperature
26	MonitorRad4	Radial coordinate of the 4th monitor temperature
27	MonitorAxi5	Axial coordinate of the 5th monitor temperature
28	MonitorRad5	Radial coordinate of the 5th monitor temperature
29	MonitorAxi6	Axial coordinate of the 6th monitor temperature
30	MonitorRad6	Radial coordinate of the 6th monitor temperature
31	MonitorAxi7	Axial coordinate of the 7th monitor temperature
32	MonitorRad7	Radial coordinate of the 7th monitor temperature
33	MonitorAxi8	Axial coordinate of the 8th monitor temperature
34	MonitorRad8	Radial coordinate of the 8th monitor temperature
35	DimRad	Number of simulation points in radial direction
36	DimAxi	Number of simulation points in axial direction
36 + i	cpErde(i)	Specific heat capacity of the earth in the axial layer i
36 + i + DimAxi	rhoErde(i)	Density of the earth in the axial layer i
36+i+2* DimAxi	lambdaErde(i)	Heat conductivity of the earth in the axial layer i
37 + 3* DimAxi	R1 or Ra or Rb	R_1 if calcBTR = 2 R_b if calcBTR = 3 R_a if calcBTR = 4
38 + 3* DimAxi	Rb	R_b if calcBTR = 4

Input	Fortran variable	Description
1	Massenstrom	Total mass flow rate for both pipes together
2	TSink	Inlet temperature to the borehole (evaporator outlet temperature)

Out-put	Fortran variable	Description
1	Massenstrom	Total mass flow rate for both pipes together
2	TSource	Source temperature of the brine
3	Massenstrom *cpSole*3.6* (TSource - TSink)	Heat transfer rate out of the borehole
4	TEarth (monitor1)	1st monitor temperature
5	TEarth (monitor2)	2nd monitor temperature
6	TEarth (monitor3)	3rd monitor temperature
7	TEarth (monitor4)	4th monitor temperature
8	TEarth (monitor5)	5th monitor temperature
9	TEarth (monitor6)	6th monitor temperature
10	TEarth (monitor7)	7th monitor temperature
11	TEarth (monitor8)	8th monitor temperature

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