



## TRNSYS Type

# **Dual-stage compressor heat pump including frost and cycle losses**

Version 2.0

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Model description and implementation in TRNSYS

Developed in the project  
Low Temperature Low Cost Heat Pump Heating System  
carried out by the Information Center for Electricity Applications  
under contract of the Swiss Federal Office of Energy

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## Type204: Dual-stage compressor heat pump

### General description

The heat pump is modelled as a black-box. The model is based on the one used in the YUM simulation program [1, 2]. The boundary conditions are the evaporator and condenser inlet temperature, the evaporator and condenser mass flow and the control signal of an external simulated controller. The model allows both steady-state and transient behaviour to be simulated for dual-stage heat pumps. In addition, single-stage units can also be simulated by treating them as dual-stage heat pumps whose operation is restricted to the first level.

This mathematical model extends to multi-stage compressor heat pumps and its implementation would only require altering the number of operating levels specified in the TRNSYS module.

The power of the condenser and the evaporator is calculated based on characteristic power curves which are usually supplied by the manufacturer of the heat pump. The curves show the condenser power and the electric power as a function of the evaporator inlet temperature, the condenser outlet temperature and the operating level (see Fig. 1 for a single-stage heat pump). These values are used to calculate coefficients of biquadratic polynomials. The calculation of these coefficients has to be carried out with either the YUM simulation program or the program 'Polynom' which is an extracted part of YUM. To increase the power of the heat pump by keeping the coefficient of performance (COP) constant, the condenser and the compressor power can be linearly scaled with a constant factor.

These polynomials are valid only for steady-state conditions. To take the cycle losses of the heat pump into account, the computed power must be corrected using the solution of a first order differential equation, known in the control theory as a  $PT_1$ -element.

Power reduction due to icing and defrosting, if not already taken into account in the manufacturer's specifications, can be computed using a semi-empiric approach. However, with this model, it is not possible to

determine at which timestep the ice will be melted.

Based on validation of the YUM algorithm with measurement data the expected accuracy of the model is:

	Relative error
Condenser energy	6.6%
Compressor energy	12.5%
COP	2.7%

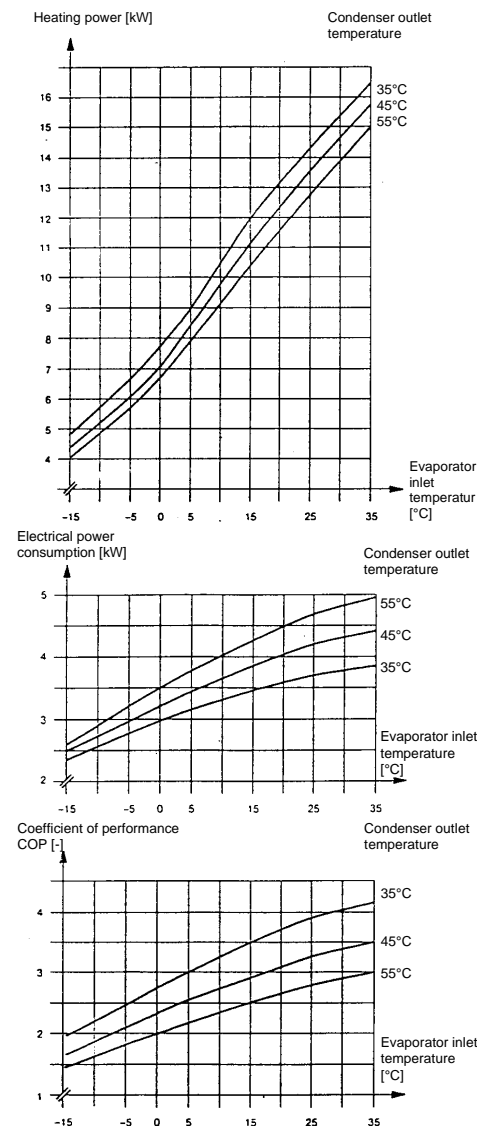


Fig. 1: Power characteristics of a single-stage heat pump.

## Symbols

### Variables

$\tau$	Time constant
$\Delta$	Difference
A	Constant for frost/defrost loss
B	Constant for frost/defrost loss
bp	Polynomial coefficient for compressor power
bq	Polynomial coefficient for condenser power
C	Constant for frost/defrost loss
c	Specific heat capacity
COP	Coefficient of performance
D	Constant for frost/defrost loss
E	Constant for frost/defrost loss
f	Scaling factor for heat pump power
m	Mass flow
P	Electrical power
Q	Heating power
T	Temperature
t	time

### Indices

c	Condenser
corr	Corrected
e	Evaporator
f	Fictitious
hp	Heat pump
ice	Icing/defrosting
icycle	Including cycle losses
in	Inlet
init	Initial
lb	Lower boundary
m	Mean value
n	Normalized
nom	Nominal value
off	Off
ol	operating level
on	On
out	Outlet
plug	Power at the electrical terminals
ss	Steady state
ub	Upper boundary
wol	Without losses

## Mathematical description

Sign convention: added power or energy is always positive, emitted always negative.

### Steady state condenser and compressor power

The biquadratic polynomial coefficients corresponding to each operating level of the heat pump must be determined in advance, using the external program 'Polynom' or YUM. After reading in each set of coefficients, they are multiplied by a scaling factor:

$$bq_{ol,i} = f bq_{ol,i}$$

$$bp_{ol,i} = f bp_{ol,i}$$

$$\text{for } ol = 1, 2 \text{ and } i = 1 \dots 6$$

Eq. 1

The steady-state power is then computed with the appropriate biquadratic polynomials for the current operating level:

$$Q_{ss,c,wol} = bq_{ol,1} + bq_{ol,2}T_{n,e,in} + bq_{ol,3}T_{n,c,out} + bq_{ol,4}T_{n,e,in}T_{n,c,out} + bq_{ol,5}T_{n,e,in}^2 + bq_{ol,6}T_{n,c,out}^2$$

Eq. 2

$$P_{ss,plug} = bp_{ol,1} + bp_{ol,2}T_{n,e,in} + bp_{ol,3}T_{n,c,out} + bp_{ol,4}T_{n,e,in}T_{n,c,out} + bp_{ol,5}T_{n,e,in}^2 + bp_{ol,6}T_{n,c,out}^2$$

Eq. 3

In the polynomial, normalized temperatures according to the formula

$$T_n = \frac{T [^\circ\text{C}]}{273.15} + 1.0$$

Eq. 4

are used.

### Iteration of condenser outlet temperature

The condenser outlet temperature is used as an independent variable in Eq. 2 and Eq. 3. Since the condenser outlet temperature is also dependent of the result of Eq. 2 and Eq. 3, it must be calculated iteratively. The iteration is carried out with the Van Wijngaarden-Decker-Brent algorithm [3]. This algorithm combines the stability of the bisection

with the calculation speed of the inverse quadratic interpolation.

## Cycling losses

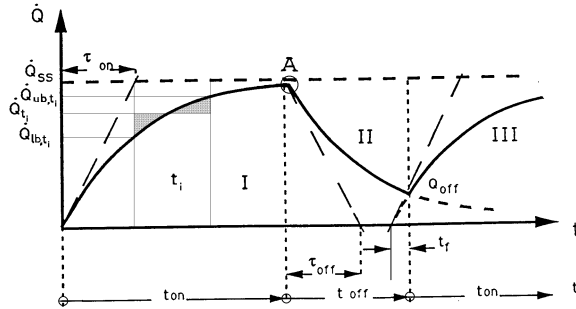


Fig. 2: Cycling losses shown with an example of a discrete time step.

After the heat pump is switched on, the machine has to be heated up and the pressure difference between the evaporator and the condenser must be built up, resulting in a power reduction during start-up. The transient power reduction during the heating-up process of an initially cold heat pump (area I) can be described using an exponential transient

$$\Delta \dot{Q}_{on,c} = \dot{Q}_{ss,c} e^{-\frac{t_{on}}{\tau_{on,ol}}} \quad \text{Eq. 5}$$

where the time constant  $\tau_{on,ol}$  may depend on the operating level of the heat pump. The resulting effective condenser power (without icing and defrosting losses) is then:

$$\begin{aligned} \dot{Q}_c &= \dot{Q}_{ss,c} - \Delta \dot{Q}_{on,c} \\ &= \dot{Q}_{ss,c} \left( 1 - e^{-\frac{t_{on}}{\tau_{on,ol}}} \right) \end{aligned} \quad \text{Eq. 6}$$

In the event that the heat pump has not cooled down completely, the switch-on time can be offset by a time shift  $t_f$  according to Fig. 2 (area III), and Eq. 6 can therefore be written as:

$$\begin{aligned} \dot{Q}_c &= \dot{Q}_{ss,c} - \Delta \dot{Q}_{on,c} \\ &= \dot{Q}_{ss,c} \left( 1 - e^{-\frac{t_f + t_{on}}{\tau_{on,ol}}} \right) \end{aligned} \quad \text{Eq. 6}$$

In terms of the time shift  $t_f$ , the effective condenser power at the moment the heat pump is switched on is given by:

$$\dot{Q}_{lb,c} = \dot{Q}_{ss,c} \left( 1 - e^{-\frac{t_f}{\tau_{on,ol}}} \right) \quad \text{Eq. 7}$$

Eq. 7 can be solved to obtain the time shift  $t_f$  in terms of the condenser power  $\dot{Q}_{lb,c}$ :

$$t_f = -\tau_{on,ol} \ln \left( 1 - \frac{\dot{Q}_{lb,c}}{\dot{Q}_{ss,c}} \right) \quad \text{Eq. 8}$$

The condenser power (Eq. 6) can be specified using the time shift  $t_f$  from Eq. 8, or, alternatively, it can be specified directly in terms of  $\dot{Q}_{lb,c}$ :

$$\dot{Q}_c = \dot{Q}_{ss,c} - \left( \dot{Q}_{ss,c} - \dot{Q}_{lb,c,init} \right) e^{-\frac{t_{on}}{\tau_{on,ol}}} \quad \text{Eq. 9}$$

where  $\dot{Q}_{lb,c,init}$  denotes the initial value of  $\dot{Q}_{lb,c}$  at the moment the heat pump is switched on (border of area II and area III). In contrast to Eq. 8, this expression remains valid even if  $\dot{Q}_{ss,c} < \dot{Q}_{lb,c}$ .

Whenever the heat pump is switched off, its energy is assumed to decrease exponentially, and the initial value of the cooling-down function needs to be known (Fig. 2, point A). Consequently, the power at the upper boundary of the current time interval must be continually re-calculated during the operating phase using the expression

$$\dot{Q}_{ub,c} = \dot{Q}_{ss,c} + \left( \dot{Q}_{lb,c,init} - \dot{Q}_{ss,c} \right) e^{-\frac{t_{on}}{\tau_{on,ol}}} \quad \text{Eq. 10}$$

In the event that the heat pump is switched off, the corresponding cooling-down curve in Fig. 2 (area II) is calculated analogously to the start-up transient previously described:

$$\dot{Q}_{loss,c} = \dot{Q}_{ss,c,nom} e^{-\frac{t_f + t_{off}}{\tau_{off}}} \quad \text{Eq. 9}$$

The cooling-down process is assumed to be proportional to the nominal power of the heat pump (at 7°C evaporator inlet temperature, 35°C condenser outlet temperature and the maximum operating level). Therefore, the time constant for the cooling-down process which is derived from

measurement data has to be based on this nominal power.

The time shift  $t_f$  is calculated analogously to that in Eq. 8, but with a decreasing exponential function:

$$\dot{Q}_{lb,c} = \dot{Q}_{ss,c,nom} e^{-\frac{t_f}{\tau_{off}}} \quad \text{Eq. 10}$$

$$t_f = -\tau_{off} \ln\left(1 - \frac{\dot{Q}_{lb,c}}{\dot{Q}_{ss,c,nom}}\right) \quad \text{Eq. 11}$$

The power at the lower boundary of the interval of the current time step  $t$  is equal to the power at the upper boundary of the last time step  $t-\Delta t$ . The latter is already computed with Eq. 9.

Therefore, the cycle loss at the upper boundary of the current time interval is given by

$$\dot{Q}_{ub,c} = \dot{Q}_{ss,c,nom} e^{-\frac{t_f + t_{ub}}{\tau_{off}}} \quad \text{Eq. 12}$$

where  $t_{ub}$  is the difference between the upper boundary of the current time interval and the shut-down time.

This value will be used if the heat pump is switched on again in the next time step.

The mean condenser power over the current time step is calculated using the integral of the power (Eq. 9) over the time step

$$\begin{aligned} \dot{Q}_{m,c} &= \frac{1}{\Delta t} \int_{t_{lb}}^{t_{ub}} \dot{Q}_c dt \\ &= \dot{Q}_{ss,c} + \frac{\tau_{on,ol}}{\Delta t} (\dot{Q}_{ss,c} - \dot{Q}_{lb,c,init}) \left( e^{-\frac{t_{ub}}{\tau_{on,ol}}} - e^{-\frac{t_{lb}}{\tau_{on,ol}}} \right) \end{aligned} \quad \text{Eq. 13}$$

The COP is therefore, taking the cycling losses into account:

$$COP_{icycle} = \frac{-\dot{Q}_{m,c}}{P_{plug}} \quad \text{Eq. 14}$$

## Icing and defrosting of the evaporator

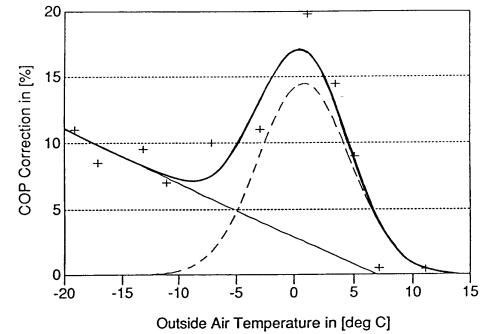


Fig. 3: COP reduction due to icing and defrosting of the evaporator. (Dots indicate measurement data)

The relative variation of the COP due to frosting and defrosting of the evaporator is described by a modified Gauss curve [4] (see Fig. 3).

The curve results from a superposition of a Gauss curve with a straight line. The Gauss approximation represents the maximal frost occurrence between 0°C and +5°C (high absolute air humidity). The straight line stands for the energy input for defrosting, which increases with decreasing outside air temperature. This energy is used for heating up the metal of the evaporator, the refrigeration in the evaporator and the heating up and melting of the ice.

The relative variation of the COP can therefore be calculated according to:

For  $A + BT_{e,in} > 0$ :

$$\Delta COP_{ice} = A + BT_{e,in} + Ce \frac{(T_{e,in} - D)^2}{E}$$

For  $A + BT_{e,in} \leq 0$

$$\Delta COP_{ice} = Ce \frac{(T_{e,in} - D)^2}{E}$$

Eq. 15

The COP in consideration of all losses (cycle loss, icing and defrosting) can be computed with:

$$COP_{corr} = COP_{icycle} (1 - \Delta COP_{ice})$$

Eq. 16

## Condenser and evaporator power

With the corrected coefficient of performance  $COP_{corr}$ , the condenser and evaporator power can be calculated according to

$$\dot{Q}_{m,c} = - COP_{corr} P_{plug} \quad \text{Eq. 17}$$

$$\text{For } -(\dot{Q}_{m,c} + P_{comp}) > 0$$

$$\dot{Q}_{m,e} = -(\dot{Q}_{m,c} + P_{comp})$$

$$\text{For } -(\dot{Q}_{m,c} + P_{comp}) \leq 0$$

$$\dot{Q}_{m,e} = 0 \quad \text{Eq. 18}$$

Finally, the outlet temperature of the condenser and evaporator can be computed with:

$$T_{c,out,corr} = T_{c,in} - \frac{\dot{Q}_{m,c}}{\dot{m}_c c_c} \quad \text{Eq. 19}$$

$$T_{e,out,corr} = T_{e,in} - \frac{\dot{Q}_{m,e}}{\dot{m}_e c_e} \quad \text{Eq. 20}$$

## Heat pump mode

The variable  $hpmode$  is set on the output #12. It is an indicator that shows in which mode the heat pump is currently operating. The following modes are possible:

hpmode	Description
100	Heat pump on, usual operation
200	Heat pump switched off due to signal from external controller (yhp=0)
210	Low-pressure error. Evaporator inlet temperature lower than low-pressure thermostat
220	Low-pressure error. Evaporator outlet temperature lower than low-pressure thermostat
230	Low-pressure error. No mass flow through evaporator
250	High-pressure error. Condenser inlet temperature higher than high-pressure thermostat
260	High-pressure error. Condenser outlet temperature higher than high-pressure thermostat
270	High-pressure error. No mass flow through condenser



## Component configuration

Parameter	Fortran variable	Description	Input	Fortran variable	Description
1	scale	Scale factor for heat pump power, uniformly applied to both operating levels	19	LUNbq(1)	Logical unit number of file containing the polynomial coefficients of the condenser power corresponding to operating level 1
2	ce	specific heat of evap. fluid	20	LUNbq(2)	Logical unit number of file containing the polynomial coefficients of the condenser power corresponding to operating level 2
3	cc	specific heat of cond. fluid			
4	Pcar	power of carter heating			
5	loprth	Set point of low-pressure thermostat (temperature)	21	LUNbp(1)	Logical unit number of file containing the polynomial coefficients of the compressor power corresponding to operating level 1
6	hiprth	Set point of high-pressure thermostat (temperature)			
7	airhp	Flag for evaporator icing/defrosting (0: No icing/defrosting is calculated, 1: Icing/defrosting is calculated)	22	LUNbp(2)	Logical unit number of file containing the polynomial coefficients of the compressor power corresponding to operating level 2
8	COPcorr1	1 <sup>st</sup> COP correction value on straight line of frost curve			
9	COPcorr2	2 <sup>nd</sup> COP correction value on straight line of frost curve			
10	COPcorr3	Maximum COP correction on Gauss curve ( <i>not</i> on the superposition of the Gauss curve and the straight line!)			
11	Tdbcorr1	Outside air temperature at 1 <sup>st</sup> COP correction value			
12	Tdbcorr2	Outside air temperature at 2 <sup>nd</sup> COP correction value			
13	Tdbcorr3	Outside air temperature at maximum of Gauss curve			
14	Tdbcorr4	Width (temperature) of the gauss curve on the half height of the Gauss maximum			
15	tauon(1)	Heat-up time constant for operating level 1, related to the mean operation power			
16	tauon(2)	Heat-up time constant for operating level 2			
17	tauoff	Cool-down constant, related to evaporator inlet temp. +7°C, condenser outlet temp. +35°C and max. operating level			
18	nchangemax	Maximal number of changes of the heat pump mode during a TRNSYS timestep			

*assign filename LUNbq(1)*  
*assign filename LUNbq(2)*  
*assign filename LUNbp(1)*  
*assign filename LUNbp(2)*  
*All files must be generated with one of the two programs YUM or Polynom. If the YUM-files are used, the first 10 rows have to be expanded to 80 character (fill in blanks).*



Out-put	Fortran variable	Description
1	mdote	Mass flow evaporator
2	Teout	Outlet temperature evaporator
3	mdotc	Mass flow condenser
4	Tcoutc	Outlet temperature condenser
5	Qdotmc	Mean condenser power over the time step
6	Qdotme	Mean evaporator power over the time step
7	Pcomp	Compressor power
8	Pcar	Carter heating power
9	(Pcomp+Pcar)	Sum of compressor and carter heating power
10	COPc	Coefficient of performance, including cycling and icing/defrost losses
11	deltCOP	Relative COP reduction due to icing/defrost losses
12	hpmode	Operation mode of heat pump
13	switch	Number of heat pump switch-ons since start of simulation
14	timeint	If the heat pump is changed from not running to running in the <i>current</i> time step: Timedifference between the last switch on signal and the current time step, otherwise: timeint = 0.

## Literature

- 1 Afjei Thomas; YUM, A Yearly Utilization Model for Calculating the Seasonal Performance Factor of Electric Driven Heat Pump Heating Systems, Technical Form; Eidgenössische Technische Hochschule Zürich, IET-LES; Zürich 1989; Schweiz
- 2 Afjei Thomas, Wittwer Dieter; Yearly Utilization Model YUM WP/Holz, Benutzerhandbuch mit Beispielen; INFEL/KRE; Zürich 1995; Schweiz
- 3 Press William H., Flannery Brian P., Teukolsky Saul A., Vetterling William T.; Numerical Recipes, The Art of Scientific Computing; ISBN 0 521 30811 9; Cambridge University Press; Cambridge MA 1987; USA
- 4 Conde M. R.; Progress Report IEA-Annex 10, Air-to-Water Heat Pump, Simple Simulation Model; Eidgenössische Technische Hochschule Zürich, IET-LES; Zürich 1985; Schweiz