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Simulation Model

Finned Water-to-Air Coil without Condensation

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Note:

This model will be a part of the HVAC component and system library for the SPARK simulation program.

The library is currently under development.

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Abstract

A simple simulation model of a finned water-to-air coil without condensation is presented. The model belongs to a collection of simulation models that allows efficient computer simulation of heating, ventilation, and air-conditioning (HVAC) systems. The main emphasis of the models is short computation time and use of input data that are known in the design process of an HVAC system. The target of the models is to describe the behavior of HVAC components in the part load operation mode, which is becoming increasingly important for energy efficient HVAC systems. The models are intended to be used for yearly energy calculation or load calculation with time steps of about 10 minutes or larger. Short-time dynamic effects, which are of interest for different aspects of control performance, are neglected.
The part load behavior of the coil is expressed in terms of the nominal condition and the dimensionless variation of the heat transfer with change of mass flow and temperature on the water side and the air side. The effectiveness-NTU relations are used to parametrize the convective heat transfer at nominal conditions and to compute the part load conditions. Geometrical data for the coil are not required. The calculation of the convective heat transfer coefficients at nominal conditions is based on the ratio of the air side heat transfer coefficients multiplied by the fin efficiency and divided by the water side heat transfer coefficient. In this approach, the only geometrical information required are the cross section areas, which are needed to calculate the fluid velocities. The formulas for estimating this ratio are presented. For simplicity the model ignores condensation.

The model is static and uses only explicit equations. The explicit formulation ensures short computation time and numerical stability. This allows using the model with sophisticated engineering methods such as automatic system optimization. The paper fully outlines the algorithm description and its simplifications. It is not tailored for a particular simulation program to ensure easy implementation in any simulation program.

Introduction

Most water-to-air coil simulation models are based on input data that are hard to obtain by the HVAC system designer, such as the fin spacing. The models are usually developed for research work rather than for system design and most of them are rather complex, with only few that have been broken down into the most important laws that describe their physical behavior accurately enough for system design.

The available simple models for water-to-air coils usually do not take the dependence of the convective heat transfer coefficient on the air mass flow and temperature into account. The more detailed models that address this dependence require geometrical knowledge of the exchanger, which is often not available during the design process of an HVAC system ([Brandemuehl et.al. 93], [TRNSYS 96]).

The model that we have developed describes the steady-state part load behavior using a dimensionless variation of the sensible heat transfer at nominal conditions. The air side and water side heat transfer coefficients at nominal conditions are computed based on nominal inlet mass flows and temperatures, the air outlet temperature and the ratio of the air side heat transfer coefficient times the fin efficiency divided by the water side heat transfer coefficient. To minimize energy loss, this ratio should be unity. But for cost reasons unity might not be achieved. The ratio can be determined from detailed calculations of the heat transfer coefficient or from an approximation based on curve fit done by Holmes [Holmes 82].

The dependence of the convective heat transfer coefficient on the mass flow variation and temperature variation is taken into account for both fluids.

An iteration is only required during the model initialization if the model is used as a cross flow heat exchanger with both streams unmixed. For all other flow configurations, no iteration is required. The numerical solution has to be done only once during the whole simulation. Convergence of the numerical solution is guaranteed.

General Description

The model represents the static behavior of a finned heating or cooling coil. Water circulates through the tubes and air passes over the finned outside of the tubes. Condensation is neglected. The main purpose of this model is to calculate the yearly energy consumption of an HVAC system.

Since condensation is ignored it should be used with care in climates where the dew point temperature of the air frequently drops below the coil surface temperature.

The input to the model are the air-side and water-side inlet mass flows and temperatures, the heat transfer coefficient, and the ratio between the $(h \times A)$ values of the water side and air side, all at the nominal operating point. No geometrical data are used. The model computes the outlet temperature on both water and air side as a function of the inlet conditions. The dependence of the convective heat transfer coefficient on the fluid velocity and temperature is taken into account for both fluids.

Simplifications

- static model
- no condensation
- fouling neglected
- thermal resistance of the heat exchanger material neglected
- fin efficiency independent of capacity flow
- no heat loss to the environment

Abbreviation

Variables

e	exchanger heat transfer effectiveness
J	temperature
r	density
h	efficiency
m	dynamic viscosity
n	kinematic viscosity
x	small positive number (ξ<<1)
A	area
C	capacity rate
c	specific heat capacity
c	constant
d	diameter
h	convective heat transfer coefficient
k	thermal conductivity
m	mass flow
n	exponent for air side heat transfer coefficient
NTU	number of exchanger heat transfer units
Nu	Nusselt number
Q	heat transfer rate
R	thermal resistance
r	ratio of heat transfer
Re	Reynolds number
s	relative sensitivity
U	heat transfer coefficient
V	velocity
x	factor for thermal variation of fluid properties
Z	capacity rate ratio

Subscripts

0	nominal (design) point
a	air
avg	average
f	fin
i	inner
in	inlet
max	maximum
min	minimum
out	outlet
r	fin root
w	water

Mathematical Description

Exchanger Heat Transfer Effectiveness

The dimensionless exchanger heat transfer effectiveness, *e*, is defined as the actual heat transfer divided by the maximum possible heat transfer [ASHRAE 85]:

$$e = \frac{\dot{Q}}{\dot{Q}_{\max}}$$

Eq. 1

If the heat loss of the heat exchanger to the environment is neglected and no phase change occurs, the heat balance of the fluid streams can be written as

$$\dot{Q} = \dot{C}_w \left(J_{w,in} - J_{w,out}\right) = \dot{C}_a \left(J_{a,out} - J_{a,in}\right)$$

Eq. 2

where C stands for the capacity flow

$$\dot{C} = \dot{m}\, c_p$$

Eq. 3

The maximum heat exchange is given by the product of the lower capacity flow and the inlet temperature difference, i.e.,

$$\dot{Q}_{\max} = \dot{C}_{\min} \left|J_{w,in} - J_{a,in}\right|$$

Eq. 4

with

$$\dot{C}_{\min} = \min\left(\dot{C}_a, \dot{C}_w\right)$$

Eq. 5

Substituting Eq. 2 and Eq. 4 into Eq. 1, the exchanger heat transfer effectiveness can be computed by

$$\boxed{e = \frac{\dot{C}_a \left(J_{a,in} - J_{a,out}\right)}{\dot{C}_{\min} \left(J_{a,in} - J_{w,in}\right)}}$$

Eq. 6

and similarly

$$\boxed{e = \frac{\dot{C}_w \left(J_{w,in} - J_{w,out}\right)}{\dot{C}_{\min} \left(J_{w,in} - J_{a,in}\right)}}$$

Eq. 7

Number of Exchanger Heat Transfer Units

The effectiveness can also be expressed as a function of the Number of Heat Transfer Units, *NTU*, the capacity rate ratio, *Z*, and the flow arrangement over the heat exchanger:

$$e = f\left(NTU, Z, \; flow\ arrangement\right)$$

Eq. 8

with the dimensionless capacity rate ratio

$$Z = \frac{\dot{C}_{\min}}{\dot{C}_{\max}}$$

Eq. 9

and the dimensionless Number of Transfer Units

$$NTU = \frac{U_{avg}\; A}{\dot{C}_{\min}}$$

Eq. 10

Table 1 lists ***e**-NTU* relations for different flow arrangements. Single-row heating and cooling coils can be considered to be cross flow heat exchangers with C_{max} mixed and C_{min} unmixed. Coils with two or more rows can be considered to be counter flow heat exchangers. Experiments have shown that these two equations are sufficient to define the performance of typical coils [Holmes 82].

counter flow (Coil with two or more rows)	$e(Z \neq 1) = \dfrac{1-e^{-NTU\,(1-Z)}}{1-Z\;e^{-NTU\,(1-Z)}}$ $\lim\limits_{Z\to 1}\left[\dfrac{1-e^{-NTU\,(1-Z)}}{1-Z\;e^{-NTU\,(1-Z)}}\right] = \dfrac{1}{1+NTU^{-1}}$ Eq. 11 Possible range: $0 \leq e \leq 1$	$NTU(Z \neq 1) = \dfrac{1}{Z-1}\ln\left(\dfrac{1-e}{1-e\;Z}\right)$ $\lim\limits_{Z\to 1}\left[\dfrac{1}{Z-1}\ln\left(\dfrac{1-e}{1-e\;Z}\right)\right] = \dfrac{e}{1-e}$ Eq. 12
parallel flow	$e = \dfrac{1-e^{-NTU\,(1+Z)}}{1+Z}$ Eq. 13 Possible range: $0 \leq e \leq \dfrac{1}{1+Z}$	$NTU = -\dfrac{\ln(-e-e\;Z+1)}{Z+1}$ Eq. 14
cross flow, both streams unmixed	$e = 1-\exp\left(\dfrac{e^{-NTU\;Z\,h}-1}{Z\,h}\right)$ $with\ h = NTU^{-0.22}$ Eq. 15 Possible range: $0 \leq e \leq 1$	$NTU = f(e, NTU, Z)$ Eq. 16 must be solved numerically. However, the solution is unique (see [Wetter 98]).
cross flow (single pass), C_{max} mixed and C_{min} unmixed. (Coil with one row)	$e = \dfrac{1-\exp\left(-Z\left(1-e^{-NTU}\right)\right)}{Z}$ $\lim\limits_{NTU\to\infty} e = \dfrac{1-e^{-Z}}{Z}$ Eq. 17 Possible range: $0 \leq e \leq \dfrac{1-e^{-Z}}{Z}$ $\lim\limits_{Z\to 0}\dfrac{1-e^{-Z}}{Z} = 1$ and $0 \leq e\;Z \leq 1-e^{Z\,(x-1)}$ (derivation see below)	$NTU = -\ln\left(1+\dfrac{\ln(1-e\;Z)}{Z}\right)$ $\lim\limits_{Z\to 0} NTU = -\ln(1-e)$ Eq. 18
for all configurations	$\lim\limits_{Z\to 0} e = 1-e^{-NTU}$ Eq. 19	

Table 1: Equations for the exchanger heat transfer effectiveness, e, and its inverse for NTU for different heat exchanger configurations (Eq. 11, see [Kays, London 84], Eq. 13 see [Holman 76], Eq. 15 see [Incropera, DeWitt 90], Eq. 17 see [Incropera, DeWitt 90])

As shown later, NTU_0 is calculated in that model as a function of e_0, which is computed from Eq. 6.

If the user enters wrong input values for Eq. 6, e_0 might have a value that cannot be obtained with the selected flow arrangement. In this case the logarithms in the equations for NTU_0 as a function of e_0 could become undefined. It is recommended that these wrong inputs be detected by checking if e_0 is inside the bounds listed in Table 1 before proceeding with the NTU_0 calculations.

Since there are two logarithms in Eq. 18, we have to examine the feasible range of the product $e \times Z$ in more detail. The argument of the outer logarithm is allowable if

$$1+\frac{\ln(1-e\ Z)}{Z} \geq x, \quad x << 1$$

Eq. 20

Since the capacity rate, Z, is always non-negative, this inequality can be written in the form

$$\ln(1-e\ Z) \geq Z\ (x-1)$$

Eq. 21

And, after exponentiating both sides and isolating $e \times Z$, we get

$$e\ Z \leq 1-e^{Z\ (x-1)}$$

Eq. 22

From the definition of Z and x, the right hand side is always smaller than unity (but still bigger than zero). Therefore, satisfying Eq. 22 automatically ensures that both of the logarithms in Eq. 18 are defined.

Eq. 15 is exact only for Z = 1, but can be used for $0 < Z \leq 1$ as an excellent approximation [Incropera, DeWitt 90].

Note that Eq. 15 can not be solved analytically for *NTU*. However, as showed in [Wetter 98], the solution for *NTU* is unique if it is written in the form

$$1-\exp\left(\frac{e^{-NTU^{0.78}\ Z}-1}{Z\ NTU^{-0.22}}\right)-e=0$$

Eq. 23

and solved for *NTU* using an algorithm such as Regula Falsi or Bisection. The efficiency of the algorithm is not critical since Eq. 23 has to be solved only once during the whole simulation.

Outlet Temperatures

If the heat exchanger effectiveness is known, we can compute the outlet temperature of both streams by using Eq. 6 or Eq. 7 respectively, which gives

$$\boxed{J_{a,out} = J_{a,in} - e\,\frac{\dot{C}_{\min}}{\dot{C}_a}\left(J_{a,in} - J_{w,in}\right)}$$

Eq. 24

and

$$\boxed{J_{w,out} = J_{w,in} + e\,\frac{\dot{C}_{\min}}{\dot{C}_w}\left(J_{a,in} - J_{w,in}\right)}$$

Eq. 25

Heat Transfer

The Number of Transfer Units depends on the product of the heat exchanger area and the overall coefficient of heat transfer from fluid to fluid, $(U_{avg}\times A)$.

For a finned pipe, $(U_{avg}\times A)$ can be written as

$$\left(U_{avg}\ A\right) = \frac{1}{\left(\frac{1}{h\ A}\right)_w + \left(\frac{1}{U\ A}\right)_{pipe} + \left(\frac{1}{h\ A}\right)_a^*}$$

Eq. 26

where $(1/(h\cdot A))_a{}^*$ stands for the thermal resistance from the air-side pipe surface to the air. Hence, it consists of the thermal resistance of the fins and the convective heat transfer from the fin surface to the air and from the pipe surface to the air, i.e.,

$$\left(\frac{1}{h\ A}\right)_a^* = f\left(\left(\frac{1}{U\ A}\right)_f,\ \left(\frac{1}{h\ A}\right)_a\right)$$

Eq. 27

The thermal resistance of the convective heat transfers is much bigger than the thermal resistance of the pipe, i.e.,

$$\left(\frac{1}{U\ A}\right)_{pipe} << \left(\frac{1}{h\ A}\right)_w + \left(\frac{1}{h\ A}\right)_a^*$$

Eq. 28

Therefore, the resistance of the pipe can be neglected, leading to

$$\left(U_{avg}\ A\right) = \frac{1}{\left(\frac{1}{h\ A}\right)_w + \left(\frac{1}{h\ A}\right)_a^*}$$

Eq. 29

In the steady state, the heat transfer between the root of the fin and the air is

$$Q = \left(h\ A\right)_a^*\left(J_r - J_a\right)$$

Eq. 30

The heat transfer from the fin surface to the air can be calculated according to

$$Q = \int h_a\left(J_f - J_a\right)dA$$

Eq. 31

where h_a is the convective heat transfer coefficient from the fin surface to the air and J_f the local fin temperature.

The fin efficiency, h_f, is defined as the quotient of the heat transferred from the fin to the air divided by the heat that would have been transferred if the whole fin were at its root temperature, i.e.,

$$h_f = \frac{\int h_a\left(J_f - J_a\right)dA}{\int h_a\left(J_r - J_a\right)dA}$$

Eq. 32

Using Eq. 30, we can express the thermal resistance from the fin root to the air by

$$\frac{1}{(h\,A)_a^*} = \frac{(J_r - J_a)}{Q}$$

Eq. 33

Substituting Q in Eq. 33 with Eq. 31 leads to

$$\frac{1}{(h\,A)_a^*} = \frac{(J_r - J_a)}{\int h_a\,(J_f - J_a)\,dA}$$

Eq. 34

The divisor of the right hand side of Eq. 34 can be substituted using the definition of the fin efficiency (Eq. 32). Assuming that h_a is constant over the whole fin, we get

$$\begin{aligned}\frac{1}{(h\,A)_a^*} &= \frac{(J_r - J_a)}{h_f \int h_a\,(J_r - J_a)\,dA} \\ &= \frac{(J_r - J_a)}{h_f\; h_a\,(J_r - J_a)\int dA} \\ &= \frac{1}{h_f\,(h\,A)_a}\end{aligned}$$

Eq. 35

Now, we can rewrite Eq. 29 by using Eq. 35, which gives

$$\boxed{(U_{avg}\;A) = \left(\frac{1}{(h\,A)_w} + \frac{1}{h_f\,(h\,A)_a}\right)^{-1}}$$

Eq. 36

The $(h{\times}A)$ values can usually not be determined, unless the geometry of the heat exchanger is known. However, to determine the $(h{\times}A)_0$ values at the nominal operating point, we can use the $(U_{avg}{\times}A)_0$ value calculated from NTU_0 (Eq. 10, at the nominal point). The NTU_0 value itself depends only on the inlet condition (temperature and mass flow) of both streams, the required heat transfer rate, Q_0, and the flow arrangement. These data are already known in the design process since they are required to specify the heat exchanger.

Ratio of Convective Heat Transfer

To determine both heat transfer rates, $(h{\times}A)_{i,0}$, we only have to know either one of them or their ratio, r. We define the ratio of the convective heat transfers by

Definition:

$$r = \frac{h_f\,(h\,A)_a}{(h\,A)_w}$$

Eq. 37

Solving Eq. 37 for the dividend and inserting into Eq. 36 (solved for the required convective heat transfer on the water side) gives

$$\boxed{(h\,A)_{w,0} = (U_{avg}\;A)_0\,\frac{r+1}{r}}$$

Eq. 38

And, similarly, for the air side:

$$\boxed{h_f\,(h\,A)_{a,0} = r\,(h\,A)_{w,0}}$$

Eq. 39

One goal of heat exchanger design is to ensure that the convective heat transfer is similar on both sides. However, r is a user input since $r = 1$ is usually not possible for cost reasons. This makes the model more flexible, particularly if more detailed knowledge about the convective heat transfer coefficient is known.

There are different ways to calculate r. Determining $(h{\times}A)_{i,0}$ from geometrical data is usually not possible in early design. However, if one knows the convective heat transfer on one side and the $(U_{avg}{\times}A)$ value, then r can be computed easily by combining Eq. 36 and Eq. 37, which gives

$$r = \frac{(U_{avg}\;A)}{(h\;A)_w - (U_{avg}\;A)}$$

Eq. 40

or

$$r = \frac{h_f\,(h\;A)_a - (U_{avg}\;A)}{(U_{avg}\;A)}$$

Eq. 41

Another way of calculating r is to use an approximation for both convective heat transfers. Holmes lists some approximation formulas for the thermal resistance of heating and cooling coils [Holmes 82]. He did a curve fit of the thermal resistance of different coils. The thermal resistance per row is described by

$$R = a_1 V_a^{-0.8} + a_2 V_w^{-0.8} + a_3$$
$$[R] = \frac{m^2\, K}{kW}; \quad [V] = \frac{m}{s}$$

Eq. 42

where V_a is the face air velocity of the heat exchanger and the reference surface of the thermal resistance is the exchanger face area. Typical values for the constants are shown in Table 2.

	a_1	a_2	a_3
Heating coil			
Low efficiency (low number of fins, no agitators)	1.32	0.44	0.49
Nominal coil (low number of fins, agitator)	1.1	0.2	0.38
High efficiency (more fins, agitators)	0.68	0.2	0.38
Cooling coil			
High fin spacing	1.025	0.208	0.326

Table 2: Coefficients for approximating the thermal resistance of different coils [Holmes 82]

Since the coefficients of Eq. 42 are obtained by curve fitting, the coefficient a_3 might also contain part of the convective thermal resistance, even though a_3 is independent on the fluid velocity. The coefficients a_1 and a_2 clearly describe the dependence of the corresponding fluid flow only.

The convective heat transfer for the air and water side can, therefore, be written as

$$\frac{1}{h_f\,(h\,A)_{a,0}} = \frac{1}{A}\left(a_1\, V_{a,0}^{-0.8} + f_1(a_3)\right)$$

Eq. 43

and

$$\frac{1}{(h\,A)_{w,0}} = \frac{1}{A}\left(a_2\, V_{w,0}^{-0.8} + f_2(a_3)\right)$$

Eq. 44

where the unknown functions f_1 and f_2 indicate that part of the constant a_3 might be attributed to the convective heat transfer.

Assuming that the convective heat transfers depend mainly on turbulence, which is described by the fluid velocity, and depend only weakly on heat diffusion, we can neglect the constant terms in the convective heat transfer. Thus, for an approximation to the convective heat transfer ratio we get

$$r \approx \frac{a_2}{a_1}\left(\frac{V_{a,0}}{V_{w,0}}\right)^{0.8}$$

Eq. 45

Fig. 1 shows r according to Eq. 45 for a heating coil at nominal conditions as a function of the two air velocities.

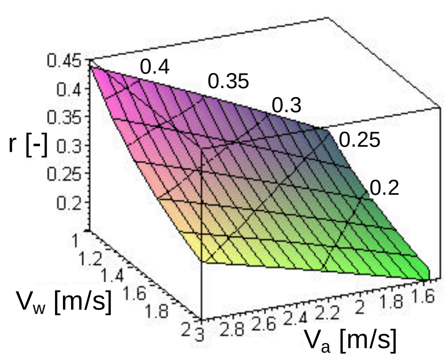

Fig. 1: Ratio of convective heat transfers, r, for a heating coil at nominal conditions.

Setting f_1 and f_2 to zero introduces an error in r. Fig. 2 shows how the selection of r affects the exchanger effectiveness and the transferred heat. In this case both mass flows are varied separately from 100% to 10%. The chart shows the transferred heat, Q, for r = 0.25, divided by the heat that would have been transferred for r = 0.50 or 0.75. It also shows the efficiency, e, for all three values of r.

The model parameters have following values:

r	0.25/0.50/0.75	
flow arrangement	1 (counter flow)	
$m_{a,0}$	0.4981	kg/s
$\vartheta_{a,in,0}$	5	°C
$\vartheta_{a,out,0}$	25	°C
$m_{w,0}$	0.2391	kg/s
$\vartheta_{w,in,0}$	40	°C
n	0.8	
m_a	0.4981 to 0.04981	kg/s
$\vartheta_{a,in}$	5	°C
m_w	0.2391 to 0.02391	kg/s
$\vartheta_{w,in}$	40	°C

Table 3: Parameter and input values for the simulation shown in Fig. 2

The selected values of r differ by a factor of two to three in order to show the extreme case if one chooses a grossly incorrect value of r. Despite the large difference in the values of r, reducing the air mass flow to 10% of its nominal value gives a change in Q value of less than 8%. Reducing the water mass flow to 10% gives a change in Q of 10% to 20% in this test case.

Therefore, even a really bad value of r only effects the heat transfer at the 10% level. It can be shown that if both mass flows are reduced by the same amount, the ratio r does not affect the variation of the overall $(U_{avg} \times A)$ value, provided that the exponents of the mass flow ratio are equal.

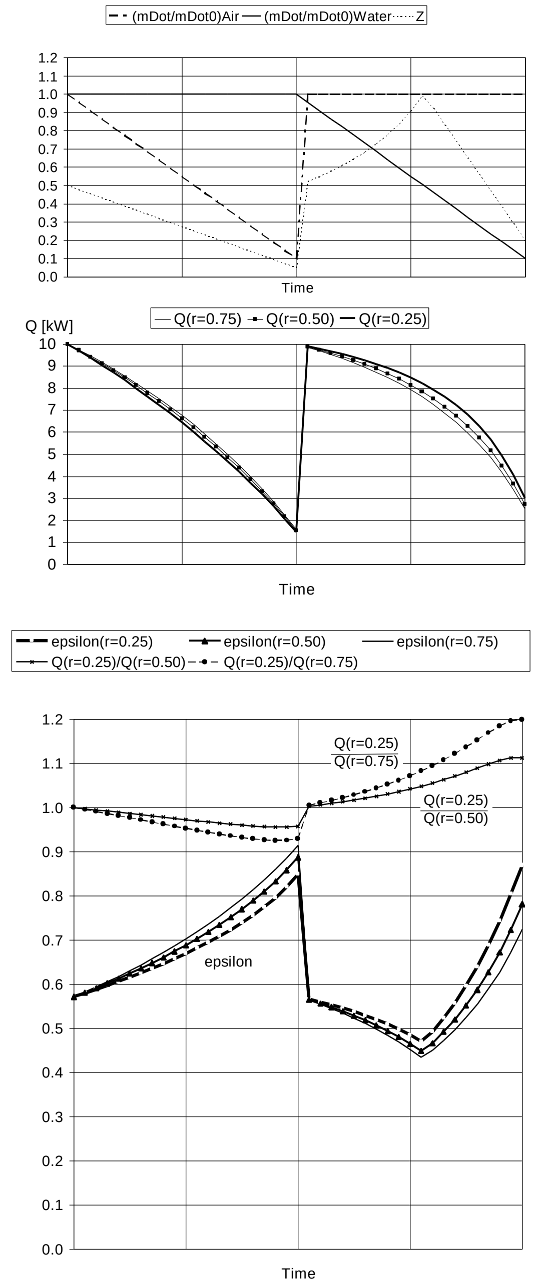

Fig. 2: Influence of the ratio, r, on the exchanger effectiveness and the heat transfer by varying mass flows

Variation of the Convective Heat Transfer Coefficients

The convective heat transfer coefficients are most sensitive to the fluid velocity and the fluid temperature. To account for the part load behavior of the heat exchanger, the dependence of the convective heat transfer coefficients on these parameters is taken into account.

Water side

[Schack 33] discusses a comparison of five different sources of convective heat transfer coefficients for turbulent flow of water in tubes. He simplifies the different equations to

$$h_w = 1842 \frac{V_w^{0.85}}{d_i^{0.1}} \left(1 + 0.014\ J_w\right)$$

$$\left[h_w\right] = \frac{W}{m^2\ K};\ \left[V\right] = \frac{m}{s};\ \left[J_w\right] = °C$$

Eq. 46

The deviation of Eq. 46 from measurements is not larger than the one for the more complex formulas he examined. The expression is good for pipes that are not very short and for diameters up to 0.1 m.

The relative sensitivity of Eq. 46, as a function of different fluid temperatures, is

$$s = \frac{1}{h_w} \frac{\partial h_w}{\partial J_w} = \frac{0.014}{1 + 0.014\ J_w}$$

$$\left[s\right] = K^{-1}$$

Eq. 47

The relative sensitivity, *s*, varies for water temperature between 20°C and 90°C by almost a factor 2 from 0.011 to 0.006 and should therefore not be assumed to be constant (see Fig. 3).

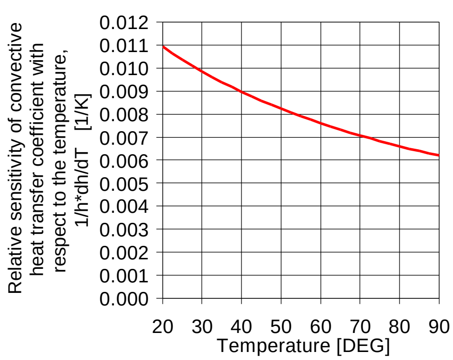

Fig. 3: Relative sensitivity, s, of water side convective heat transfer coefficient with respect to temperature.

From Fig. 3 we can say that using the mean fluid temperature, neglecting the wall temperature, is sufficiently accurate for calculating h_w. Furthermore, using the heat exchanger inlet temperature, $J_{w,in}$, instead of the local mean water temperature, is sufficiently accurate for describing the variation of the convective heat transfer coefficient. However, since the water temperature in the heat exchanger can vary over a huge range during a year, the sensitivity, *s*, should be evaluated for the current heat exchanger inlet temperature and not assumed to be constant over the yearly temperature range. Therefore, Eq. 46 is used to further describe the convective heat transfer coefficient by using the heat exchanger inlet temperature, $J_{w,in}$, instead of the local mean water temperature J_w.

For our purposes, we are interested in the variation of the convective heat transfer coefficient, $h_w/h_{w,0}$. The first-order approximation of the convective heat transfer coefficient is

$$h_w\left(J_{w,0}+\Delta J_w\right)= h_w\left(J_{w,0}\right)+s\,h_w\left(J_{w,0}\right)\left(J_w-J_{w,0}\right)+O\left(\Delta J_w{}^2\right)$$

Eq. 48

Dividing this by Eq. 46 evaluated at $J_{w,0}$ gives the relative variation of the convective heat transfer coefficient:

$$\frac{h_w\left(J_w\right)}{h_w\left(J_{w,0}\right)}=1+s\left(J_{w,0}\right)\left(J_w-J_{w,0}\right)+O\left(\Delta J_w{}^2\right)$$

Eq. 49

If the second-order influence of the water temperature is neglected and the notation

$$x_w=1+s\left(J_{w,0}\right)\left(J_w-J_{w,0}\right)$$

Eq. 50

is introduced, Eq. 49 can be written as

$$\frac{h_w\left(J_w\right)}{h_w\left(J_{w,0}\right)}=x_w\left(J_w\right)$$

Eq. 51

Fig. 4 plots this expression for a base water temperature of 40°C. In HVAC applications the temperature variation between full and part load is usually less than 20 K. Therefore, the error due to neglecting the temperature influence on the variation of the convective heat transfer coefficient is a maximum of 20%. This variation should not be neglected.

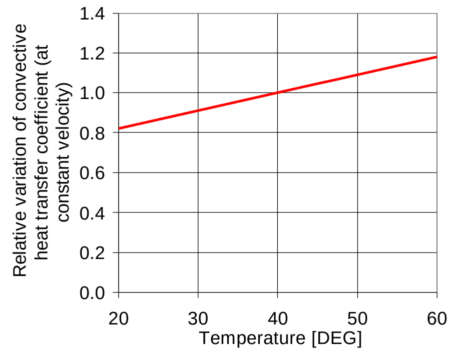

Fig. 4: Relative variation of convective heat transfer coefficient, $h_w/h_{w,0}$, at constant water velocity

The influence of the water velocity on the heat transfer coefficient can be obtained by evaluating $h_w(V)/h_w(V_o)$ using Eq. 46. This gives

$$\frac{h_w(V)}{h_w(V_0)}=\left(\frac{V}{V_0}\right)^{0.85}$$

Eq. 52

or, in terms of mass flow using

$$V=\frac{4\,\dot{m}}{r\;d_i^2\;p}$$

Eq. 53

$$\frac{h_w(V)}{h_w(V_0)}=\left(\frac{\dot{m}_w}{\dot{m}_{w,0}}\;\frac{r_{w,0}}{r_w}\right)^{0.85}$$

Eq. 54

Since the variation of the water density is less than 2.5% over a temperature change of 20 K, this density variation can be neglected in Eq. 54. Therefore, the variation of the water side convective heat transfer coefficient can finally be written as

$$\boxed{\frac{h_w}{h_{w,0}}=x_w\left(J_w\right)\left(\frac{\dot{m}_w}{\dot{m}_{w,0}}\right)^{0.85}}$$

Eq. 55

Air Side

[Stasiulevicius, Skrinska 87] compared correlations for the convective heat transfer coefficient of finned tube bundles. Generally, the correlation can be written in the form

$$Nu = c\,\mathrm{Re}^{n}$$

Eq. 56

where c is a constant that depends on the geometry of the heat exchanger only.

To calculate the Nusselt number we have to distinguish bundles of tubes that are in line *vs.* staggered bundles.

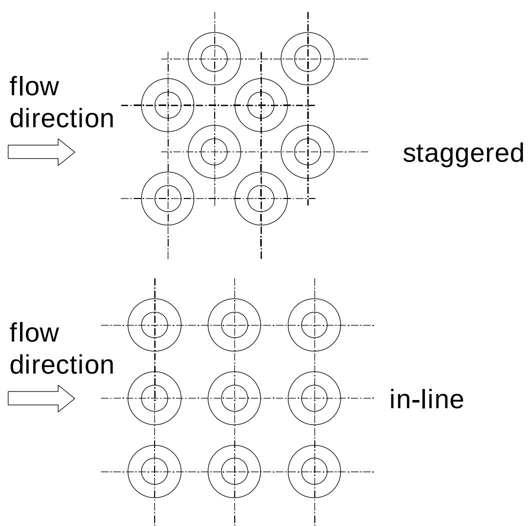

Fig. 5: Staggered and in-line bundles

For *in-line bundles*, [Yudin et. al., 68] suggest the correlation of the form of Eq. 56 with $n = 0.72$, where the physical properties are determined by the mean temperature. The fin pitch was used as the reference dimension in *Re* and *Nu*. The flow velocity is determined at the narrowest cross section of the bundles. The correlation is valid only for $Re \le 4{\cdot}10^4$. No correlation is known for in-line bundles with $Re > 4{\cdot}10^4$.

For *staggered bundles*, Stasiulevicius and Skrinska divide the calculation into three areas of different *Re*: (I) a mixed flow zone ($Re = 1{\cdot}10^3$ to $2{\cdot}10^4$), (II) a zone where turbulent flow starts predominating in the mixed boundary layer ($Re = 2{\cdot}10^4$ to $2{\cdot}10^5$) and (III) a zone with developed turbulent boundary flow ($Re > 2{\cdot}10^5$).

The exponent for calculating *Nu* is given in Table 4. The reference dimension for *Nu* and *Re* is the fin pitch.

tube arrange-ment		
in-line	$\mathrm{Re} \le 4{\cdot}10^4$	n = 0.72
staggered	$\mathrm{Re} = 1{\cdot}10^3$ to $2{\cdot}10^4$	n = 0.65
staggered	$\mathrm{Re} = 2{\cdot}10^4$ to $2{\cdot}10^5$	n = 0.8
staggered	$\mathrm{Re} \ge 2{\cdot}10^5$	n = 0.95

Table 4: Exponent of Reynolds number for calculating Nusselt number

Using Eq. 56, the variation of the air-side Nusselt number can be written as

$$\frac{Nu_a}{Nu_{a,0}} = \left(\frac{\mathrm{Re}_a}{\mathrm{Re}_{a,0}}\right)^{n}$$

Eq. 57

If the Nusselt number is replaced with

$$Nu_a = \frac{h_a\,d}{k_a}$$

Eq. 58

and the Reynolds number by

$$\mathrm{Re}_a = \frac{V_a\,d}{n_a}$$

Eq. 59

the variation of the convective heat transfer coefficient becomes

$$\frac{h_a}{h_{a,0}} = \frac{k_a}{k_{a,0}} \left(\frac{V_a}{V_{a,0}} \, \frac{n_{a,0}}{n_a} \right)^n$$

Eq. 60

The velocity, V_a, in Eq. 57 can be substituted with

$$V_a = \frac{\dot{m}_a}{r_a \, A}$$

Eq. 61

and the kinematic viscosity, n_a, by the dynamic viscosity, m_a, and the density, r_a,

$$n_a = \frac{m_a}{r_a}$$

Eq. 62

which gives

$$\frac{h_a}{h_{a,0}} = \frac{k_a}{k_{a,0}} \left(\frac{\dot{m}_a}{\dot{m}_{a,0}} \, \frac{m_{a,0}}{m_a} \right)^n$$

Eq. 63

Eq. 63 can also be written in the form

$$\frac{h_a}{h_{a,0}} = x_a\left(J_a\right) \left(\frac{\dot{m}_a}{\dot{m}_{a,0}} \right)^n$$

Eq. 64

where x_a expresses the variation of the air properties as a function of the temperature, i.e.,

$$x_a\left(J_a\right) = \frac{k_a}{k_{a,0}} \left(\frac{m_{a,0}}{m_a} \right)^n$$

Eq. 65

In the following, we derive a simple expression for $x_a(J_a)$. As shown in Fig. 6, the heat conductance of dry air at a pressure of 1 bar can be approximated linearly by

$$k_a = 2.453 \cdot 10^{-2} + 7.320 \cdot 10^{-5} \; J_a$$

$$[k] = \frac{W}{m \cdot K}; \; [J] = °C$$

Eq. 66

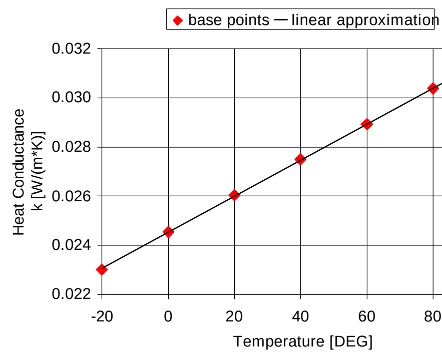

Fig. 6: Approximation of heat conductance, k, for dry air at 1 bar. Values from [Cerbe, Hoffmann 90]

For the dynamic viscosity, m_a, the linear approximation for dry air at 1 bar is (see Fig. 7)

$$m_a = 1.706 \cdot 10^{-5} + 4.529 \cdot 10^{-8} \; J_a$$

$$[m] = \frac{kg}{m \cdot s}; \; [J] = °C$$

Eq. 67

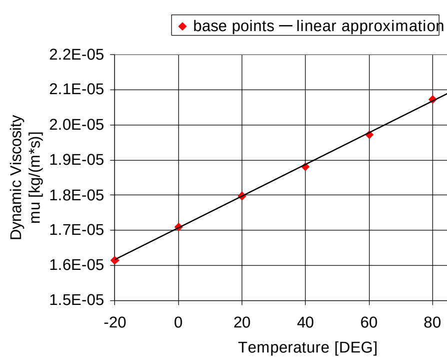

Fig. 7: Approximation of dynamic viscosity, m, for dry air at 1 bar. Values from [Cerbe, Hoffmann 90]

Using these approximations, the ratio x_a can be expressed as a function of the air temperature, J_a, and the exponent, *n*.

As shown in Fig. 8, x_a can be approximated by a first-order Taylor series expansion with respect to the variable *J*, about the temperature J_0. A nominal value of 25°C is selected for J_0. This is in the middle of the range for cooling and heating applications. As can be seen in Fig. 8, the original form of x_a is almost linear and there is not much dependence on the exponent *n* (for *n* in the range of 0.65 to 0.95). Therefore, (I) the derivative $dx_a/d\,J_a$ is assumed to be constant over the whole range of J_a, and (II) the Taylor expansion, evaluated for *n* = 0.8, is used for all values of *n*. This leads to approximating x_a with

$$x_a = 1 + 4.769 \cdot 10^{-3} \left(J_a - J_{a,0}\right) + O\left(\left(J_a - J_{a,0}\right)^2\right)$$

Eq. 68

where the second-order terms will be neglected.

The maximum relative error of Eq. 68 is less than 5% in the range of J = -20 to 100°C and *n* = 0.65 to 0.95, as can also be seen in Fig. 8. The biggest deviation is for temperatures above 60°C, which are not encountered in most HVAC systems.

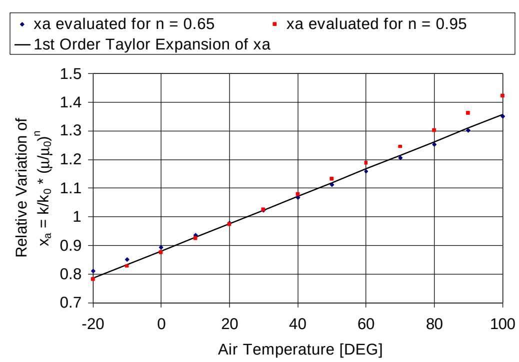

Fig. 8: Relative variation of x_a, with nominal temperature $J_{a,0}$ = 25°C.

To avoid an iteration over the heat exchanger, the factor x_a that represents the air property variation is evaluated using the air inlet temperature and not the mean air temperature.

Using the approximation Eq. 68 for the variation of the air properties, the variation of the air-side convective heat transfer coefficient is

$$\frac{h_a}{h_{a,0}} = x_a \left(\frac{\dot{m}_a}{\dot{m}_{a,0}} \right)^n$$

Eq. 69

Overall Heat Transfer

To compute the overall heat transfer coefficient, $(U_{avg} \times A)$, Eq. 69 and Eq. 55 can be inserted into Eq. 36. This gives

$$x_a = 1 + 4.769 \cdot 10^{-3} \left(J_{a,in} - J_{a,in,0} \right)$$

Eq. 70

$$\left(h_f \, (h \; A)_a \right) = x_a \left(\frac{\dot{m}_a}{\dot{m}_{a,0}} \right)^n \left(h_f \, (h \; A)_a \right)_0$$

Eq. 71

$$x_w = 1 + s\left(J_{w,in} \right)\left(J_{w,in} - J_{w,in,0} \right)$$

Eq. 72

$$\left(h_w \; A \right) = x_w \left(\frac{\dot{m}_w}{\dot{m}_{w,0}} \right)^{0.85} \left(h_w \; A \right)_0$$

Eq. 73

and finally

$$\left(U_{avg} \; A \right) = \left(\frac{1}{(h \; A)_w} + \frac{1}{h_f \, (h \; A)_a} \right)^{-1}$$

Eq. 74

Now, all values for calculating the heat exchanger behavior at any mass flow and temperature are known.

Interface

The interface to the model is separated into input and output variables. However, in some simulation languages ([SPARK 97], [NMF 96]) the user can specify which variables are inputs and which are outputs.

Parameter

No.	Variable	Description
1	*r*	ratio between air-side and water side convective heat transfer coefficient (Eq. 37)
2	-	flow arrangement: 1: counter flow (Coil with two or more rows) 2: parallel flow 3: cross flow, both streams unmixed 4: cross flow (single pass), C_{max} mixed and C_{min} unmixed. (Coil with one row)
3	$m_{a,0}$	air mass flow at nominal operating point
4	$J_{a,in,0}$	air inlet temperature at nominal operating point
5	$J_{a,out,0}$	air outlet temperature at nominal operating point
6	$m_{w,0}$	water mass flow at nominal operating point
7	$J_{w,in,0}$	water inlet temperature at nominal operating point
8	*n*	exponent for air side heat transfer coefficient

Input

No.	Variable	Description
1	m_a	air mass flow
2	$J_{a,in}$	air inlet temperature
3	m_w	water mass flow
4	$J_{w,in}$	water inlet temperature

Initial value

none

Output

No.	Variable	Description
1	m_a	air mass flow
2	$J_{a,out}$	air outlet temperature
3	m_w	water mass flow
4	$J_{w,out}$	water outlet temperature
5	e	exchanger heat transfer effectiveness
6	Q	transferred heat

Algorithm

This section shows conceptually how the derived formulas can be used in a sequential algorithm.

Initalization (executed only once during the simulation)

$\varepsilon_0 \leftarrow C_{a,0},\ C_{w,0},\ \vartheta_{a,in,0},\ \vartheta_{a,out,0},\ \vartheta_{w,in,0}$ (Eq. 6)

iterate $NTU_0 \leftarrow NTU_0,\ \varepsilon_0,\ Z_0$ (Eq. 12, Eq. 14, Eq. 16, or Eq. 18)

$(U_{avg}\cdot A)_0 \leftarrow NTU_0,\ C_{min}$ (Eq. 10)

$(h\cdot A)_{w,0} \leftarrow r,\ (U_{avg}\cdot A)_0$(Eq. 38)

$(\eta_f\cdot(h\cdot A)_{a,0}) \leftarrow r,\ (h\cdot A)_{w,0}$ (Eq. 39)

store $(h\cdot A)_{w,0},\ (\eta_f\cdot(h\cdot A)_{a,0}),\ m_{a,0},\ m_{w,0},\ \vartheta_{a,in,0},\ \vartheta_{w,in,0}$

Each call

$x_a \leftarrow \vartheta_{a,in},\ \vartheta_{a,in,0}$ (Eq. 70)

$(\eta_f\cdot(h\cdot A)_a) \leftarrow x_a,\ m_a/m_{a,0},\ n,\ (\eta_f\cdot(h\cdot A)_{a,0})$ (Eq. 71)

$s \leftarrow \vartheta_{w,in}$ (Eq. 47)

$x_w \leftarrow \vartheta_{w,in},\ \vartheta_{w,in,0}$ (Eq. 72)

$(h\cdot A)_w \leftarrow m_w/m_{w,0},\ x_w,\ \vartheta_{w,in},\ \vartheta_{w,in,0}$ (Eq. 73)

$(U_{avg}\cdot A) \leftarrow (\eta_f\cdot(h\cdot A)_a),\ (h\cdot A)_w$ (Eq. 74)

$C_{min} \leftarrow C_a,\ C_w$ (Eq. 5)

$NTU \leftarrow (U_{avg}\cdot A),\ C_{min}$ (Eq. 10)

$\varepsilon \leftarrow NTU,\ Z$ (Eq. 11, Eq. 13, Eq. 15, or Eq. 17)

$\vartheta_{a,out}\ \vartheta_{w,out} \leftarrow \varepsilon,\ \vartheta_{a,in},\ \vartheta_{w,in},\ C_a,\ C_w$ (Eq. 24 and Eq. 25)

$Q \leftarrow C_a,\ \vartheta_{a,out},\ \vartheta_{a,in}$ (Eq. 2)

Conclusion

Using the effectiveness-NTU relation and the ratio $r = h_f\times(h\times A)_a/(h\times A)_w$ at the nominal operating point, the convective heat transfer coefficients times the exchanger surfaces (and times the fin efficiency for the air side) at nominal condition can be determined.

Describing the dependence of the convective heat transfer as a power function of the water and air velocity allows the part-load performance of the coil to be calculated and eliminates the need for geometrical information. In this approach, the outlet condition is expressed as an explicit function of the inlet mass flows and temperatures only. The inputs to the model are data that are known during the design process of an HVAC system, namely, the inlet mass flows and temperatures and the air outlet temperature at the nominal operating point.

Formulating the model with explicit rather than implicit equations ensures short computation times and avoids convergence problems.

Acknowledgments

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